GAMING UNDER UNCERTAINTY: A BAYESIAN MULTISTAGE BARGAINING MODEL WITH COMPUTER-SIMULATED DECISION-MAKING PROCESSES

By

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Any errors in this dissertation are my responsibility.

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GAMING UNDER UNCERTAINTY: A BAYESIAN MULTISTAGE BARGAINING MODEL WITH COMPUTER-SIMULATED DECISION-MAKING PROCESSES

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This dissertation proposes a two-person multistage bargaining process in the context of a bilateral monopoly's vertical pricing strategies in which the two "Bayesian" decision agents consistently revise their prior probabilistic conjectures during the pricing process. In order to examine the equilibrium points and bargaining behavior in this multistage pricing game, a computer program was written to simulate and determine the monopolist's and the monopsonist's optimal pricing strategies. This simulated bargaining model has shown the following results:

(1) The limiting equilibrium point can not be reached because the variance in each player's prior probability density function is never equal to zero. But by defining some threshold for these variances, the limiting equilibrium point exists in this model.

(2) By adding certain constraints (the minimum profit) in this model, the bargaining process will end when these conditions are satisfied. Under certain circumstances, if information can be exchanged completely and if each player's profit share is mutually acceptable, the bargaining game may end up with a vertical integrated market.

This paper, combining the multistage game with computer simulation, provides a research direction toward a unified approach to dealing with bargaining problems.

CHAPTER I
INTRODUCTION

Two-Person Gaming Problems

Two-person gaming decisions are interesting and important in many business and economic problems, but finding a satisfactory way to solve these problems is not easy. The difficulty here is how to model the problems such that the "solutions" are sufficiently realistic as to reflect real-world gaming behavior. While most previous research in two-person game theory focused on deterministic models (Luce, 1957; Rapoport, 1966; Cross, 1969; Shubik, 1975; Young, 1975), recent research has formulated problems with "incomplete information" such that each player was making "probabilistic conjectures" to his opponent's reactions before taking his own actions (Chatterjee and Samuelson, 1983; Corstjens and Horowitz, 1985).

Corstjens and Horowitz's paper (1985) provided an important step in solving the two-person non-zero-sum gaming problem in the context of bilateral monopoly's vertical pricing strategies. But their one-stage gaming model was not able to reflect the "interactive" nature inherent in a bargaining process. This paper, based on Corstjens and Horowitz's model (1985), proposes a multistage bargaining process in which the two "Bayesian" decision agents consistently revise their probabilistic conjectures in successive pricing stages. In order to examine whether any equilibrium points exist in this multistage bargaining model, a computer program is written to simulate and determine the monopolist's and the monopsonist's optimal pricing strategy. This approach of combining the multistage game with computer simulation provides a new research direction in studying the human bargaining and market behavior (Shubik and Levitan, 1980).

The Bilateral Trading Problem

The bilateral trading problem (Corstjens and Horowitz, 1985) in this paper is the following: A manufacturer sells one product with fixed unit cost c to a retailer who will resell this product to consumers with known market demand. It is assumed both the manufacturer and the retailer have the full knowledge of the total cost function $C = cq$ and the market demand function $q = aP^{\varepsilon}$, where P is the market price and $\varepsilon < -1$ is the demand elasticity. Each decision agent's task is to maximize his own profit at the retail price of P_R and the quantity purchases of q.

Because of the full knowledge of the cost function and market demand, the manufacturer and the retailer know each other's payoff functions. On the next page in Figure 1-1, assume (1) the market demand is the curve DD, (2) the manufacturer's average = marginal cost function is the straight line CC, and (3) the manufacturer and the retailer

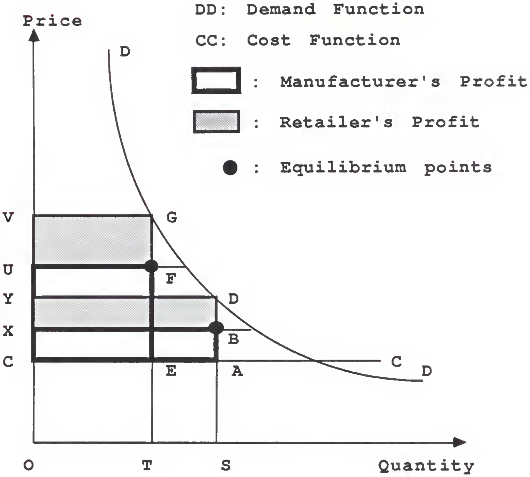

Figure 1-1. The profit share of the bilateral trading.

agree on a retail price of X and a sales quantity of S. Then, from the demand curve with q = S the manufacturer knows the retailer intends to charge the consumers the market price Y so will make profit XBDY. Similarly, form the cost function the retailer will also figure out that the manufacturer's profit is CABX under the retail price X. All the acceptable agreements for both players are in the region above the cost function and the left side of demand curve.

From Figure 1-1, this bilateral trading is a two-person non-zero-sum gaming problem in which the two players can freely accept or reject any proposed agreements, and the actions they take jointly may (1) increase both's profit, (2) decrease both's profit, (3) increase his own profit but decrease the other's profit, and (4) decrease his own profit but increase the other's profit. Each player may use any strategy (cheating, threatening, persuading) to achieve the goal of maximizing his own profit, but each realizes that there are aspects of cooperation as well as competition in this game. Generally speaking, the bilateral trading is not just a mathematical maximization problem, its nature is more like a psychological bargaining decision. Some game-theoretic bargaining papers in the negotiation process and the strategies of conflict can be found in Zartman (1978) and Schelling (1960).

Without further assumptions, it seems impossible to tell where the final agreement will be in this bilateral trading problem. The reason is in this model both players have perfect information about each other's profit (by the full knowledge of the cost function and market demand), unless both players agree with the profit they are sharing, no one can force the other to accept any agreement.

Basically, the final agreement in the bilateral trading reflects the relative bargaining (monopoly) power for the monopolist and the monopsonist in this market. If the manufacturer is in a better bargaining position in terms of personal factors and the game-situation, then we expect he will ask for more profit than the retailer does and the final profit-share will reflect the relative strength of his bargaining power. On the other hand, if the retailer is in a better bargaining position, then he will ask for more profit than the manufacturer does and the final profit-share will reflect the relative strength of his bargaining power too. The player's relative bargaining power will depend on personal factors and the game-situation. Without assumptions about these factors, it seems impossible to predict the final bargaining result.

Nash (1950) based on idealized assumptions of players' equal bargaining skills and full knowledge of each other's tastes and preferences, came out with "equal share" as the solution to the bargaining problems. But this approach does not seem apt for this problem. The reason is if one person makes some assumptions to get a solution, then the other person can make different assumptions to obtain a solution too! As a matter of fact, too many factors that can affect a bargaining result and usually any two players are different in personality, bargaining skills, and risk preferences. More importantly, most bargaining games are played with incomplete information about many crucial factors of the game situation (Harsanyi, 1967, 1968a, 1968b).

Gaming Under Uncertainty

Information plays a key role in almost every decision-making problem. For the two-person bargaining problems, the crucial information varies in different games depending on how the problems are modeled. In Harsanyi's paper (1967, 1968a, 1968b) each player's payoff functions, their strategies, and the information about the game were the crucial parameters and not precisely known by each other. In Chatterjee and Samuelson's model (1983) the minimum selling price and maximum buying price were the crucial parameters in a two-person bidding model in which the two agents maximized the expected profit over the crucial parameters' subjective probability distributions.

Basically, in a game with "incomplete information", we first decide the crucial parameters in the game and assign the subjective probability distributions to these parameters, then formulate the player's optimal strategies with subjective probability distributions. The key point here is how to decide the crucial parameters in the bargaining problems. Most papers in the bilateral trading games were focused on the payoff (preference) functions, cost functions, and demand functions as the crucial parameters with incomplete information (Samuelson, 1980; Chatterjee and Ulvila, 1982; Chatterjee and Samuelson, 1983; Crocker, 1983; Riordan, 1985).

With the "probabilistic conjectures" approach, Corstjens and Horowitz (1985) formulated the bilateral trading problem differently. In their model, they assumed both players had full knowledge of the cost function and market demand, under this game-situation, each player's price-reaction elasticity became the crucial parameter and was assigned a subjective probability density to reflect the uncertainty associated with it. Each player, the manufacturer and the retailer, then formulated his objective function with the subjective probability density to find the optimal strategy in an one-shot pricing game.

The way that Corstjens and Horowitz (1985) handled the bilateral trading problem provided an important step in solving the two-person bargaining problems. Firstly, with the full knowledge of each other's preference (payoff) functions, the two players are indeed playing a psychological bargaining game. Secondly, each player's subjective probability density represents not only the uncertainty in the bilateral trading, but also the subjective relative bargaining position in this game. For instance, if the manufacturer thinks he is in a stronger bargaining position in terms of a certain game-situation, then the subjective probability density which he assigns to the reaction elasticity will reflect these conjectures. Therefore we don't have to specify the player's personal factors or make assumptions about the game-situation. Each player's conjectures about these factors are now represented by one single parameter (the reaction elasticity) and the subjective probability density assigned to it. So Nash's idealized assumptions of players' equal bargaining skills and full knowledge of each other's tastes and preferences (1950) becomes a special case in Corstjens and Horowitz's model (1985). The classical approach (not the probabilistic conjectures) to the bilateral trading problem can be found in Deschamps and Jaskold-Gabszewicz (1975) and Scherer (1980). The related literature in the bilateral relationship and contracts can be found in Zusman and Etgar (1981), Riordan (1984, 1985), and Shugan (1985).

Quirmbach (1986) proposed a deterministic analytical model to discuss the price changes in vertical integration. Cowley (1986) set up a regression model to study relations between business margins and the buyer/seller power. Though the measurement of buyer and seller power is an interesting research topic, in this dissertation the conjectures about the opponent's reactions have been represented by the probability density assigned to the decision agent's price-reaction elasticity. The factors for assessing the probability density are not specified because these factors vary in different contexts. Ross and Krausz's paper (1986) provided a nonparametric time series test with data collected from industries to examined the price paths on the recessions and initial year of recovery. Quirmbach (1986), Cowley (1986), and Ross and Krausz's paper (1986) were somewhat irrelevant to this dissertation which is a theoretical two-person gaming model with probabilistic price-reaction conjectures.

Simulated Bargaining Processes

If the bilateral trading rule is that both the manufacturer and the retailer can only quote the retail price P_R and the purchasing quantity q once, then each one of them may take the strategy to find one P_R and one q to maximize their *expected* profit respectively. But in the real world, bargaining is seldom just an one-shot decision process. Instead, most bargaining problems are multistage *learning* processes in which each player collects information and revises his judgment until the final agreement is reached.

The information revision process is interesting and important in a bargaining game with incomplete information. Though there are more questions than answers in how people actually do update their subjective probabilities (Kahneman, Slovic, and Tversky, 1981; Hogarth, 1980), this paper adopts Bayesian theorem (Corstjens and Horowitz, 1985, p. 5) as the players' information revision method through the entire bargaining process. The multistage bargaining game as pointed out by Shubik and Levitan (1980, p. 38) is a useful analytical tool to examine the bargaining processes.

The computer-simulated bargaining process is an important feature that each player's optimal solutions is solved by a computer package (IMSL LIBRARY, 1984). Sobel and Takahashi (1983) presented an analytical multistage bargaining model but with no computer simulation. The blending of a multistage process and the computer simulation provides an interesting research direction in solving economic problems that involve the market behavior of discrepant agents. With the simulation program, we can input a variety of initial subjective probabilities to examine equilibrium points and conduct the sensitivity analysis with different market demands and pricing processes. By the simulated multistage gaming process, this paper can be a starting point toward a unified approach to dealing with bargaining problems (Shubik and Levitan, 1980, p. 4).

Outline of This Paper

In Chapter II, the bargaining model, the manufacturer and the retailer's optimal strategies, and the information revision processes are constructed. In Chapter III, the important properties of the players' pricing strategies are discussed in detail. In Chapter IV, we analyze some interesting computer output and the sensitivity analysis with respect to the market demand and different pricing processes will also be discussed. In Chapter V the further research directions in the bilateral gaming problems are addressed.

CHAPTER II
THE BAYESIAN BARGAINING PROCESS

The Model

With the "probabilistic conjectures" approach discussed in Chapter I, during the bilateral bargaining process, the manufacturer who is uncertain about the retailer's purchasing quantity q and the retailer who is uncertain about the manufacturer's retail price P_R formulate so-called quasi–Cournot conjectures (Seade, 1980; Perry and Porter, 1985) to each other's price-quantity and quantity-price reactions. Then the elasticity of the price-quantity (or quantity–price) reaction function is chosen as the crucial parameter in this bargaining "game" and assigned a subjective probability density to reflect the uncertainty of the situation. By the assumption that the manufacturer's total cost function, $C = cq$, and the market demand function, $q = aP^{\varepsilon}$, are known to both players, knowing the purchasing quantity q is equivalent to knowing the market price P. For the sake of convenience, in this paper the two players' reaction functions are formulated with the retail price, P_R, and the market price, P, as arguments. With this setting, the manufacturer's price-reaction elasticity is defined as $\theta = [(dP/P)/(dP_R/P_R)]$ and the retailer's price-reaction elasticity is defined as $\rho = [(dP_R/P_R)/(dP/P)]$, respectively.

Given above definitions, we can derive the manufacturer's implied price-reaction function $F(P_R)$ and the retailer's implied price-reaction function $G(P)$, where

$$F(P_R) \equiv P = k_1 P_R^{\theta}, \ k_1 > 0,$$

$$G(P) \equiv P_R = k_2 P^{\rho}, \ k_2 > 0.$$

For any feasible agreement the market price P must be greater than the retail price P_R, that is $P > P_R$, and for analytical purposes we can assume $k_1 = k_2 = 1$. Then the properties of the price-reaction functions can be summarized in the following:

For the Manufacturer

1) If $P_R > 1$ and $k_1 = 1$, then θ must be greater than 1.
2) If $P_R < 1$ and $k_1 = 1$, then θ must be less than 1.

For the Retailer

1) If $P_R > 1$ and $k_2 = 1$, then ρ must be less than 1.
2) If $P_R < 1$ and $k_2 = 1$, then ρ must be greater than 1.

In this paper we set the unit cost $c = 1$. It does not matter what the unit cost c will be, it can always be "normalized" to one unit of cost ($c = 1$). Because the manufacturer's retail price P_R must be greater than the unit cost c, we have $P_R > 1$ in this model. One example of the manufacturer's price-reaction function $P = P_R^{\theta}$ ($\theta > 1$) is plotted in Figure 2-1. One example of the retailer's price-reaction function $P_R = P^\rho$ ($\rho < 1$) is plotted in Figure 2-2. The "effective" reaction functions in Figure 2-1 and Figure 2-2 are in the region of $P_R > 1$ and $P > P_R$.

From the manufacturer's price-reaction function $P = P_R{}^\theta$, with P_R as a given, the less the reaction elasticity θ ($\theta > 1$), the less the market price P. If $\theta = 1$, then the retailer has no monopoly power in this bargaining game because now the market price P is equal to the retail price P_R. From the retailer's price-reaction function $P_R = P^\rho$, with P as a given, the greater the reaction elasticity ρ ($0 < \rho < 1$), the greater the retail price P_R. If $\rho = 1$, the retailer has no monopoly power in this bargaining game because now the retail price P_R is equal to the market price P.

The domains of θ and ρ are important in selecting the probability distributions. In this paper the probability distributions of θ and ρ are chosen to be the normal distributions. The main reason to choose the normal distribution is analytical tractability and the fact that with the normal distribution it is easy to provide a good approximation in the event that the actual density is symmetric. The only problem in using the normal distribution is the negative side of the distribution (both θ and ρ) must be positive. It will be seen later that this model works well by properly assigning the mean μ and the variance σ^2 in a normal distribution $N(\mu, \sigma^2)$. The variances of these subjective probabilities must be small for both players to

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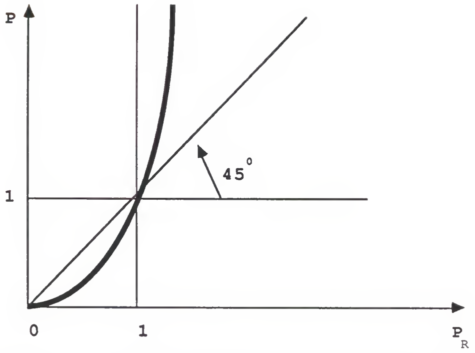

Figure 2-1. The manufacturer's price-reaction function.

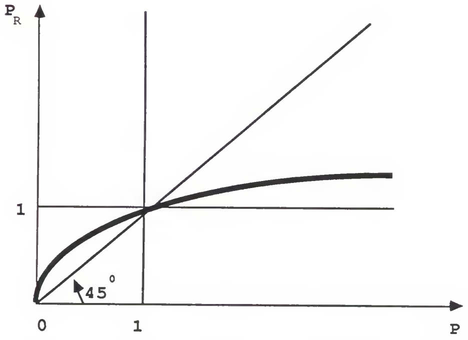

Figure 2-2. The retailer's price-reaction function.

produce a reasonable range of the retail price P_R and the market price P (this will be discussed later in this chapter). The examples for each player's probability density of the price-reaction elasticity are plotted in Figure 2-3 and Figure 2-4. Be aware of the narrow shape (small variance) in each player's probability distribution.

Because in this multistage bargaining model the main issue is the two players' information revision processes in which they use Bayes' theorem to revise their prior probabilities, the conjugate properties of normal distributions (Raiffa and Schlaifer, 1961) are applied by both players. In this paper, the manufacturer first assigns a conditional normal density function $f(\theta \backslash m_\theta) = N(m_\theta, V_\theta^2)$ over the reaction elasticity θ assuming the mean m_θ is known, then assigns another normal density function $f(m_\theta) = N(\mu_\theta, v_\theta^2)$ over the expected reaction elasticity m_θ. From Bayes' theorem we have

$$f(\theta \backslash m_\theta) = N(m_\theta, V_\theta^2),$$

$$f(m_\theta) = N(\mu_\theta, v_\theta^2),$$

$$f(\theta) = \int f(\theta \backslash m_\theta) f(m_\theta)$$

$$= N(\mu_\theta, V_\theta^2 + v_\theta^2); \qquad \text{(Appendix 1)}$$

$$\text{let } \sigma_\theta^2 = V_\theta^2 + v_\theta^2,$$

$$f(\theta) = N(\mu_\theta, \sigma_\theta^2).$$

The manufacturer's density function $f(\theta) = N(\mu_\theta, \sigma_\theta^2)$ is the resultant function of $f(\theta \backslash m_\theta)$ and $f(m_\theta)$, where $\mu_\theta = m_\theta$ and

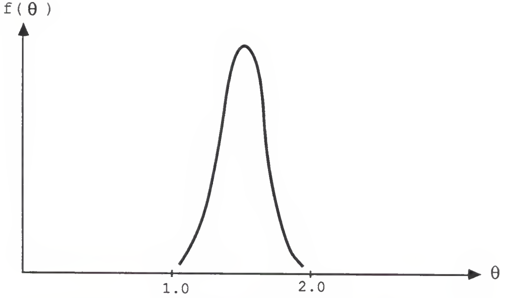

Figure 2-3. The probability density function of the manufacturer's price-reaction elasticity θ.

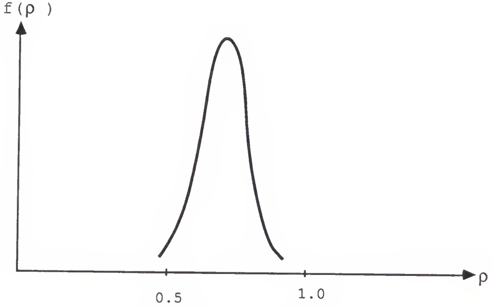

Figure 2-4. The probability density function of the retailer's price-reaction elasticity ρ.

$\sigma_\theta^2 = V_\theta^2 + v_\theta^2$. Later we will see that after each pricing stage v_θ^2 is smaller and smaller and hence that σ_θ^2 will approach V_θ^2.

The retailer also assigns a conditional normal density function $f(\rho \backslash m_\rho) = N(m_\rho, V_\rho^2)$ over the reaction elasticity ρ assuming the mean m_ρ is known, then assigns another normal density function $f(m_\rho) = N(\mu_\rho, v_\rho^2)$ over the expected reaction elasticity m_ρ. From Bayes' theorem we have

$$f(\rho \backslash m_\rho) = N(m_\rho, V_\rho^2),$$

$$f(m_\rho) = N(\mu_\rho, v_\rho^2),$$

$$f(\rho) = \int f(\rho \backslash m_\rho) f(m_\rho)$$

$$= N(\mu_\rho, V_\rho^2 + v_\rho^2); \qquad \text{(Appendix 1)}$$

$$\text{let } \sigma_\rho^2 = V_\rho^2 + v_\rho^2,$$

$$f(\rho) = N(\mu_\rho, \sigma_\rho^2).$$

The retailer's density function $f(\rho) = N(\mu_\rho, \sigma_\rho^2)$ is the resultant function of $f(\rho \backslash m_\rho)$ and $f(m_\rho)$, where $\mu_\rho = m_\rho$ and $\sigma_\rho^2 = V_\rho^2 + v_\rho^2$. It will be seen later that after each pricing stage v_ρ^2 is smaller and smaller such that σ_ρ^2 will approach V_ρ^2.

In this model, we assume both the manufacturer and the retailer are risk-neutral decision agents so their utility functions of wealth are linear functions. The main reason for assigning a linear utility function to both the manufacturer and the retailer is because we can not obtain analytical objective functions for both the manufacturer and

---

the retailer if we apply some commonly-used utility functions (Raiffa and Keeney, 1976, chapter 4) to their objective functions. The approximated objective functions fail to generate the acceptable numerical solutions because the exponential objective functions are not well represented by these approximations. For the present purpose to examine the multistage bargaining behavior, we assume both agents are risk-neutral so the risk-prone utility functions are excluded in this model.

Other important issues in this model are the pricing processes and the stopping rules in a multistage bargaining game. For a multistage bargaining process, there are three pricing processes that may occur:

1) The manufacturer and the retailer quote their retailer price P_R and market price P simultaneously in every pricing stage.
2) The manufacturer quotes the retail price P_R first and the retailer quotes the market price P next, then the process continues in this order until it stops.
3) The retailer quotes the market price P first and the manufacturer quotes P_R next, then the process continues in this order until it stops.

In this dissertation, we adopt the first pricing process that the manufacturer and the retailer quote the retail price P_R and market price P simultaneously in every pricing stage.

The impact of the second and third pricing processes to the bargaining results will be discussed and compared with the first pricing process in Chapter IV.

For the simultaneous pricing process, the interesting question is when the process stops. If this question can be answered, then the main issue in the dissertation is solved because the equilibrium point is reached when the pricing process stops. But there is no way for us to know when the bargaining process will stop unless we know the specific information about the manufacturer and the retailer's "intention" in this bargaining game. One person may be willing to accept smaller profit, if he considers himself in a weaker bargaining position. But he will ask for more profit, if he is in a stronger bargaining position. For any two persons, the bargaining results will be different. Beside the personal factors, the game-situation will also affect the bargaining results.

Let us consider the case where there are two countries dealing in bilateral trading. In this case, the profit-maximization may not be the only issue for the import country who may want to buy more quantity at a lower import price to benefit its people. Even for two profit-maximization agents, the game-situation will affect the bargaining results. It seems we are back to the problem that without further assumptions we can not predict the bargaining results, and once we make assumptions, we fall into the research approach taken in Nash's paper (1950).

One way to solve this problem may be to treat the bilateral trading problem as a "game," so the bargaining process will be stopped by some game rules. But this approach is somewhat unrealistic in that (1) the game rules are not easily made realistic, and (2) the game rules may force the decision agents to change their bargaining behavior and pricing strategies.

As a matter of fact, it is very difficult to model the real-world bargaining process by a mathematical model because there are too many factors and the game-situation must be considered. In this paper, we will not specify the stopping rules or make any assumptions to predict the bargaining results. Instead, we let the multistage bargaining process keep going to examine the trend of the decision agent's bargaining behavior. This approach has the following advantages:

(1) With the same pattern of bargaining, we can predict different equilibrium points according to any assumptions for a specified bargaining problem.
(2) Without any pre-specified assumptions, we can study the "general" bargaining behavior in the bargaining problems.

Therefore, in this paper the manufacturer and the retailer simultaneously quote P_R and P in every pricing stage, and the process is simulated on the computer in such a way that each decision agent acts like a pricing-machine. With the setting of the price-reaction function, the probability distributions of the price-reaction elasticity, and the pricing process discussed above, we can start to derive the manufacturer's and the retailer's optimal pricing strategies. For convenience, the following definitions are used in this model:

System Factors

$C = cq$	the manufacturer's cost function;
$q = aP^{\varepsilon}$	the market demand function, where $a > 0$;
c	the unit cost of the cost function, $c = 1$;
q	the quantity of the product;
ε	the market demand elasticity, where $\varepsilon < -1$;
P_R	the retail price that the manufacturer charges to the retailer, $P_R > c = 1$;
P	the market price that the retailer charges to the consumers, $P > P_R$;

For the Manufacturer

$P = k_1 P_R^{\theta}$	manufacturer's price-reaction function, where $k_1 > 0$;
θ	manufacturer's price-reaction elasticity, here $\theta > 0$ (If $k_1 = 0$, then $\theta > 1$);
$f(\theta)$	$f(\theta) = N(\mu_{\theta}, \sigma_{\theta}^2)$, a normal distribution; π_M manufacturer's profit, $\pi_M = (P_R - c)q$;
$E_{\theta}[\pi_M]$	manufacturer's expected profit over $f(\theta)$;

For the Retailer

$P_R = k_2P^{\rho}$ retailer's price-reaction function, where $k_2 > 0$;

ρ retailer's price-reaction elasticity, where $0 < \rho < 1$;

$f(\rho)$ $f(\rho) = N(\mu_{\rho}, \sigma_{\rho}^2)$, a normal distribution;

π_R retailer's profit where $\pi_R = (P - P_R)q$;

$E_{\rho}[\pi_R]$ retailer's expected profit over $f(\rho)$.

Manufacturer's Optimal Strategies

For the manufacturer with a linear utility function of wealth, maximizing the utility function is equivalent to maximizing the net profit. Under uncertainty the manufacturer decides to find a retail price P_R that maximizes his expected profit $E_{\theta}[\pi_M]$:

$$\max_{P_R} \; E_{\theta}[\pi_M] = E_{\theta}[(P_R - c)q].$$

From the demand function $q = aP^{\varepsilon}$, the manufacturer's price-reaction function $P = k_1P_R^{\theta}$, and the normal probability density $f(\theta) = N(\mu_{\theta}, \sigma_{\theta}^2)$, we obtain

$$E_{\theta}[P_R^{\varepsilon\theta}] = \exp(\varepsilon\mu_{\theta}\ln P_R + .5\varepsilon^2(\ln P_R)^2\sigma_{\theta}^2). \qquad \text{(Appendix 2)}$$

With $E_\theta[P_R^{\varepsilon\theta}]$, the manufacturer's objective function is rewritten as

$$\underset{P_R}{\text{Max}}\ E_\theta[\pi_M] = ak_1^{\varepsilon}(P_R - c)\ \exp(\varepsilon\mu_\theta \ln P_R + .5\varepsilon^2\sigma_\theta^2(\ln P_R)^2). \tag{2.1}$$

By taking the first derivative of Eq. (2.1) (defined as $E_\theta'[\pi_M]$), we have

$$E_\theta'[\pi_M] = ak_1^{\varepsilon}E_\theta[P_R^{\varepsilon\theta}]\{1 + \varepsilon\mu_\theta(1 - c/P_R) + \varepsilon^2\sigma_\theta^2 \ln P_R(1 - c/P_R)\}.$$

Letting $E_\theta'[\pi_M] = 0$, we obtain the first-order condition

$$P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0. \tag{2.2}$$

Equation (2.2) can be rewritten as

$$P_R = c(\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R)/(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R).$$

Because $P_R > c$, $(\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R)/(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R)$ must be greater than 1 such that $(\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R) < 0$ and $(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R) < 0$. By properly assigning the values to ε, μ_θ and σ_θ^2, the condition $(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R) < 0$ will hold in this model. If not, the computer output will indicate this situation. It will be seen later from the computer-generated output (Chapter III) that σ_θ^2 is very small in this model.

Letting $E_\theta''[\pi_M]$ be the second derivative with respect to P_R, we have

$$E_\theta''[\pi_M] = ak_1^{\varepsilon}E_\theta[P_R^{\varepsilon\theta}](P_R)^{-1}\{(\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R)(1 + \varepsilon\mu_\theta(1 - c/P_R) + \varepsilon^2\sigma_\theta^2 \ln P_R(1 - c/P_R)) + (\varepsilon^2\sigma_\theta^2 + c(P_R)^{-1}(\varepsilon\mu_\theta - \varepsilon^2\sigma_\theta^2 + \varepsilon^2\sigma_\theta^2 \ln P_R))\}.$$

From the first-order condition $1 + \varepsilon\mu_\theta(1 - c/P_R) + \varepsilon^2\sigma_\theta^2 \ln P_R(1 - c/P_R) = 0$, $E_\theta''[\pi_M] < 0$ can be rewritten as

$$E_\theta''[\pi_M] = \varepsilon^2\sigma_\theta^2(1 - c/P_R) + c(P_R)^{-1}(\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R) < 0.$$

If $\varepsilon^2\sigma_\theta^2(1 - c/P_R)$ is small, from the first-order condition $\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R < 0$, the second-order condition $E_\theta''[\pi_M] < 0$ will hold. If not, the computer output will indicate this situation.

If $\sigma_\theta^2 = 0$, the optimal solution under certainty is

$$P_R = c\varepsilon\mu_\theta/(1 + \varepsilon\mu_\theta). \tag{2.3}$$

Because $P_R > c$, $\varepsilon\mu_\theta/(1 + \varepsilon\mu_\theta) > 1$. With $\varepsilon < -1$, we have $\mu_\theta > -1/\varepsilon$. The properties of Eq. (2.3) are summarized as the following:

1) When $\mu_\theta \to \infty$, $\varepsilon\mu_\theta/(1 + \varepsilon\mu_\theta) \to 1$, so $P_R \to c$.
2) When $\mu_\theta \to -1/\varepsilon$, $\varepsilon\mu_\theta/(1 + \varepsilon\mu_\theta) \to \infty$, so $P_R \to \infty$.

The manufacturer's optimal retail price P_R can be obtained by solving either the maximum value of Eq. (2.1) or the root value of Eq. (2.2). The computer package used in solving the optimal retail price P_R is the International Mathematical and Statistical Library (IMSL LIBRARY, 1984).

Manufacturer's Revision of Prior Probability

Suppose at time t = 1 the manufacturer quotes the retail price $P_{R(1)}$ and the retailer quotes the market price $P_{(1)}$. From the price-reaction function $P = k_1 P_R^{\theta}$, the manufacturer defines the first observation of θ as $\theta_{(1)} = \ln(P_{(1)}/k_1)/\ln P_{R(1)}$. With the prior probability density $f(m_\theta) = N(\mu_\theta, v_\theta^2)$ and the observation $\theta_{(1)}$, the posterior probability of m_θ defined as $f_{(2)}(m_\theta \backslash \theta_{(1)})$ is derived by Bayes' rule, where

$$f_{(2)}(m_\theta \backslash \theta_{(1)}) = N\left(\frac{\theta_{(1)} v_\theta^2 + \mu_\theta V_\theta^2}{v_\theta^2 + V_\theta^2} , \frac{v_\theta^2 V_\theta^2}{v_\theta^2 + V_\theta^2} \right).$$

From $f(\theta) = \int f(\theta \backslash m_\theta) f(m_\theta)$ and $f(\theta \backslash m_\theta) = N(m_\theta, V_\theta^2)$, the posterior probability of θ given $\theta_{(1)}$ is defined as

$$f_{(2)}(\theta \backslash \theta_{(1)}) = \int f(\theta \backslash m_\theta) f_{(2)}(m_\theta \backslash \theta_{(1)}),$$

$$f_{(2)}(m_\theta \backslash \theta_{(1)}) = N\left(\frac{\theta_{(1)} v_\theta^2 + \mu_\theta V_\theta^2}{v_\theta^2 + V_\theta^2} , \frac{v_\theta^2 V_\theta^2}{v_\theta^2 + V_\theta^2} + V_\theta^2 \right).$$

The density function $f_{(2)}(m_\theta \backslash \theta_{(1)})$ is the revised prior probability function of m_θ at the pricing stage t = 2. The density function $f_{(2)}(\theta \backslash \theta_{(1)})$ is the revised resultant density function of θ at time t = 2. In general, at the end of time t, the manufacturer will have the following information:

$$\theta_{(t)} = \ln(P_{(t)}/k_1)/\ln P_{R(t)},$$

$$f_{(t+1)}(m_\theta \backslash \theta_{(t)}) = N(\mu_{\theta(t+1)}, v_{\theta(t+1)}{}^2),$$

$$\mu_{\theta(t+1)} = \frac{\theta_{(t)} v^2_{\theta(t)} + \mu_{\theta(t)} V^2_\theta}{v^2_{\theta(t)} + V^2_\theta},$$

$$v^2_{\theta(t+1)} = \frac{v^2_{\theta(t)} V^2_\theta}{v^2_{\theta(t)} + V^2_\theta} < v^2_{\theta(t)},$$

$$f_{(t+1)}(\theta \backslash \theta_{(t)}) = N(\mu_{\theta(t+1)}, \sigma_{\theta(t+1)}{}^2),$$

$$\mu_{\theta(t+1)} = \frac{\theta_{(t)} v^2_{\theta(t)} + \mu_{\theta(t)} V^2_\theta}{v^2_{\theta(t)} + V^2_\theta},$$

$$\sigma^2_{\theta(t+1)} = \frac{v^2_{\theta(t)} V^2_\theta}{v^2_{\theta(t)} + V^2_\theta} + V^2_\theta.$$

When $t \rightarrow \infty$,

$$v^2_{\theta(t+1)} \rightarrow 0, \qquad \text{(Appendix 3)}$$

$$\sigma^2_{\theta(t+1)} \rightarrow V_\theta^2.$$

Retailer's Optimal Strategies

The retailer's optimal strategy is to find a market price P which will maximize his expected profit $E_\rho[\pi_R]$. His objective function is written as

$$\max_P \; E_\rho[\pi_R] = E_\rho[(P - P_R)q].$$

Given the demand function $q = aP^\varepsilon$, the retailer's price-reaction function $P_R = k_2P^\rho$, and the normal probability density $f(\rho) = N(\mu_\rho, \sigma_\rho^2)$, we obtain

$$E_\rho[P^\rho] = \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2). \qquad \text{(Appendix 2)}$$

So the retailer's objective function becomes

$$\max_P \; E_\rho[\pi_R] = aP^{1+\varepsilon} - ak_2P^\varepsilon \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2). \qquad (2.4)$$

By taking the first derivative of Eq. (2.4) and letting $E_\rho'[\pi_R] = 0$, we obtain the first-order condition

$$(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$$
$$= 0. \qquad (2.5)$$

Because $(1+\varepsilon) < 0$ and $k_2 > 0$, the term $(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)$ must be negative in Eq. (2.5). By properly assigning the values of ε, μ_ρ and σ_ρ^2, the condition $(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P) < 0$ will hold in this model. If not, the computer output will indicate this situation.

By taking the second derivative of $E_\rho[\pi_R]$, we have

$$E_\rho''[\pi_R] = aP^{\varepsilon-2}\{\varepsilon(1+\varepsilon)P - k_2((\varepsilon-1)(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P) + \sigma_\rho^2 + (\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)(\mu_\rho + \sigma_\rho^2 \ln P))\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)\}.$$

By substituting the first-order condition $(1+\varepsilon)P = k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$ and simplifying $E_\rho''[\pi_R]$, we obtain

$$E_\rho''[\pi_R] = ak_2P^{\varepsilon-2}\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$$
$$\{(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)(1 - \mu_\rho + \sigma_\rho^2 \ln P) - \sigma_\rho^2\}.$$

With the second-order condition $E_\rho''[\pi_R] < 0$, we will have the following condition

$$(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)(1 - \mu_\rho + \sigma_\rho^2 \ln P) - \sigma_\rho^2 < 0.$$

From the first-order condition $(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P) < 0$ and $(1 - \mu_\rho + \sigma_\rho^2 \ln P) > 0$, so $(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)(1 - \mu_\rho + \sigma_\rho^2 \ln P) < 0$. Because $\sigma_\rho^2 > 0$, the second-order condition $E_\rho''[\pi_R] < 0$ holds.

The retailer's optimal solution under certainty can be derived by maximizing $\pi_R = (P - P_R)q$. Given $q = aP^\varepsilon$ and $P_R = k_2 P^\rho$, we have $\pi_R = aP^{\varepsilon+1} - ak_2 P^{\rho+\varepsilon}$. Taking the first derivative and letting it equal 0, we have

$$P = \left(\frac{\varepsilon + 1}{k_2(\varepsilon + \rho)}\right)^{\rho-1}. \tag{2.6}$$

Equation (2.6) is a special case in which the elasticity ρ is a constant. The retailer's optimal retail price P is solved by the computer package (IMSL LIBRARY, 1984) to find the maximum value of Eq. (2.4) or the root of Eq. (2.5).

Retailer's Revision of Prior Probability

Suppose at the time $t = 1$ the retailer quotes the market price $P_{(1)}$ and the manufacturer quotes the retail price $P_{R(1)}$. From the price-reaction function $P_R = k_2 P^\rho$, the retailer defines the first observation of ρ as $\rho_{(1)} = \ln(P_{R(1)}/k_2)/\ln P_{(1)}$. With the prior probability density $f(m_\rho) = N(\mu_\rho, v_\rho^2)$ and the observation $\rho_{(1)}$, the retailer's posterior probability function of m_ρ at $t = 2$, defined as $f_{(2)}(m_\rho \backslash \rho_{(1)})$, is derived by Bayes' rule, where

$$f_{(2)}(m_\rho \backslash \rho_{(1)}) = N\left(\frac{\rho_{(1)} v_\rho^2 + \mu_\rho V_\rho^2}{v_\rho^2 + V_\rho^2} , \frac{v_\rho^2 V_\rho^2}{v_\rho^2 + V_\rho^2} \right).$$

From $f(\rho) = \int f(\rho \backslash m_\rho) f(m_\rho)$ and $f(\rho \backslash m_\rho) = N(m_\rho, V_\rho^2)$, the posterior probability of ρ given $\rho_{(1)}$ is defined as

$$f_{(2)}(\rho \backslash \rho_{(1)}) = \int f(\rho \backslash m_\rho) f_{(2)}(m_\rho \backslash \rho_{(1)}),$$

$$f_{(2)}(\rho \backslash \rho_{(1)}) = N\left(\frac{\rho_{(1)} v_\rho^2 + \mu_\rho V_\rho^2}{v_\rho^2 + V_\rho^2} , \frac{v_\rho^2 V_\rho^2}{v_\rho^2 + V_\rho^2} + V_\rho^2 \right).$$

The density function $f_{(2)}(m_\rho \backslash \rho_{(1)})$ is the retailer's revised prior density function of m_ρ at the pricing stage t = 2. The density function $f_{(2)}(\theta \backslash \theta_{(1)})$ is the retailer's revised resultant density function of ρ at the pricing stage t = 2. In general, at the end of time t the retailer will have the following information:

$$\rho_{(t)} = \ln(P_{R(t)}/k_2)/\ln P_{(t)},$$

$$f_{(t+1)}(m_\rho \backslash \rho_{(t)}) = N(\mu_{\rho(t+1)}, v^2_{\rho(t+1)}),$$

$$\mu_{\rho(t+1)} = \frac{\rho_{(t)} v_{\rho(t)}^2 + \mu_{\rho(t)} V_\rho^2}{v_{\rho(t)}^2 + V_\rho^2},$$

$$v^2_{\rho(t+1)} = \frac{v^2_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}},$$

$$f_{(t+1)}(\rho \backslash \rho_{(t)}) = N(\mu_{\rho(t+1)}, \sigma^2_{\rho(t+1)}),$$

$$\mu_{\rho(t+1)} = \frac{\rho_{(t)} v^2_{\rho(t)} + \mu_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}},$$

$$\sigma^2_{\rho(t+1)} = \frac{v^2_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}} + V^2_{\rho}.$$

When $t \rightarrow \infty$,

$v^2_{\rho(t+1)} \rightarrow 0,$ (Appendix 3)

$\sigma^2_{\rho(t+1)} \rightarrow V_{\rho}^{2}.$

With the model constructed in this chapter, properties of the manufacturer and the retailer's optimal pricing strategies will be discussed in the next chapter. Understanding these important properties will help us to simulate the bilateral pricing game and examine the agent's bargaining behavior in this model.

CHAPTER III
PROPERTIES OF THIS MODEL

Equilibrium Points

Theoretically, there exists a limiting equilibrium point in this multistage bargaining model when time t goes to infinity. At time t = 1, the manufacturer starts with the following subjective Normal densities over the price-reaction elasticity θ:

$$f_{(1)}(\theta \backslash m_{\theta(1)}) = N(m_{\theta(1)}, V_\theta^2),$$
$$f_{(1)}(m_{\theta(1)}) = N(\mu_{\theta(1)}, v_{\theta(1)}^2),$$
$$f_{(1)}(\theta) = N(\mu_{\theta(1)}, \sigma_{\theta(1)}^2),$$
$$\text{where } \sigma_{\theta(1)}^2 = v_{\theta(1)}^2 + V_\theta^2.$$

During the bargaining process, with the observation $\theta_{(t)} = \ln(P_{(t)}/k_1)/\ln P_{R(t)}$ at time t, the manufacturer's subjective probability density functions at time t+1 become

$$f_{(t+1)}(\theta \backslash m_{\theta(t+1)}) = N(m_{\theta(t+1)}, V_\theta^2),$$

$$f_{(t+1)}(m_{\theta(t+1)} \backslash \theta_{(t)}) = N(\mu_{\theta(t+1)}, v_{\theta(t+1)}^2),$$

$$f_{(t+1)}(\theta \backslash \theta_{(t)}) = N(\mu_{\theta(t+1)}, \sigma_{\theta(t+1)}^2),$$

where $\sigma_{\theta(t+1)}^2 = v_{\theta(t+1)}^2 + V_\theta^2$,

$$\mu_{\theta(t+1)} = \frac{\theta_{(t)} v_{\theta(t)}^2 + \mu_{\theta(t)} V_\theta^2}{v_{\theta(t)}^2 + V_\theta^2},$$

$$v_{\theta(t+1)}^2 = \frac{v_{\theta(t)}^2 V_\theta^2}{v_{\theta(t)}^2 + V_\theta^2} < v_{\theta(t)}^2,$$

$$\sigma_{\theta(t+1)}^2 = \frac{v_{\theta(t)}^2 V_\theta^2}{v_{\theta(t)}^2 + V_\theta^2} + V_\theta^2.$$

Because $v^2_{\theta(t+1)} < v^2_{\theta(t)}$, as time $t \rightarrow \infty$, $v^2_{\theta(t+1)} \rightarrow 0$ and $\sigma^2_{\theta(t+1)} \rightarrow V_\theta^2$ (Appendix 3), so the manufacturer's probability density function at time $t \rightarrow \infty$ becomes

$$f_{(\infty)}(\theta) = N(\mu_{\theta(\infty)}, V_\theta^2).$$

The value of $\mu_{\theta(\infty)}$ depends on the limiting value of $\mu_{\theta(t+1)}$ at time $t \rightarrow \infty$. The function $f_{(t+1)}(m_{\theta(t+1)} \backslash \theta_{(t)}) = N(\mu_{\theta(t+1)}, v_{\theta(t+1)}^2)$ is no longer a probability function ($v^2_{\theta(t+1)} \rightarrow 0$) but a single real number $\mu_{\theta(\infty)}$ at time $t \rightarrow \infty$. The manufacturer, who can not revise his prior probability density function any further, will quote $P_{R(\infty)}$ (see Eq. (2.2)) as his optimal retail price, where

$$P_{R(\infty)} = c(\varepsilon\mu_{\theta(\infty)} + \varepsilon^2 V_\theta{}^2 \ln P_{R(\infty)})$$
$$(1 + \varepsilon\mu_{\theta(\infty)} + \varepsilon^2 V_\theta{}^2 \ln P_{R(\infty)})^{-1}. \qquad (3.1)$$

The value of $P_{R(\infty)}$ depends on the mean $\mu_{\theta(\infty)}$ and the variance $V_\theta{}^2$ assuming the unit cost c and the market demand elasticity ε are given.

For the retailer at time t = 1, he starts with the following subjective Normal densities over price-reaction elasticity ρ:

$$f_{(1)}(\rho\backslash m_{\rho(1)}) = N(m_{\rho(1)}, V_\rho{}^2),$$
$$f_{(1)}(m_{\rho(1)}) = N(\mu_{\rho(1)}, v_{\rho(1)}{}^2),$$
$$f_{(1)}(\rho) = N(\mu_{\rho(1)}, \sigma_{\rho(1)}{}^2),$$
$$\text{where } \sigma_{\rho(1)}{}^2 = v_{\rho(1)}{}^2 + V_\rho{}^2.$$

During the bargaining process, with the observation $\rho_{(t)} = \ln(P_{R(t)}/k_2)/\ln P_{(t)}$ at time t, the retailer's subjective probability density functions at time t+1 become

$$f_{(t+1)}(\rho\backslash m_{\rho(t+1)}) = N(m_{\rho(t+1)}, V_\rho{}^2),$$

$$f_{(t+1)}(m_{\rho(t+1)}\backslash\rho_{(t)}) = N(\mu_{\rho(t+1)}, v_{\rho(t+1)}{}^2),$$

$$f_{(t+1)}(\rho\backslash\rho_{(t)}) = N(\mu_{\rho(t+1)}, \sigma_{\rho(t+1)}{}^2),$$

$$\text{where } \sigma_{\rho(t+1)}{}^2 = v_{\rho(t+1)}{}^2 + V_\rho{}^2,$$

$$\mu_{\rho(t+1)} = \frac{\rho_{(t)} v^2_{\rho(t)} + \mu_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}},$$

$$v^2_{\rho(t+1)} = \frac{v^2_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}} < v^2_{\rho(t)},$$

$$\sigma^2_{\rho(t+1)} = \frac{v^2_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}} + V^2_{\rho}.$$

Because $v^2_{\rho(t+1)} < v^2_{\rho(t)}$, when time $t \to \infty$, $v^2_{\rho(t+1)} \to 0$ and $\sigma^2_{\rho(t+1)} \to V_\rho^2$ (Appendix 3), the retailer's probability density function at time $t \to \infty$ becomes

$$f_{(\infty)}(\rho) = N(\mu_{\rho(\infty)}, V_\rho^2).$$

The value of $\mu_{\rho(\infty)}$ depends on the limiting value of $\mu_{\rho(t+1)}$ at time $t \to \infty$. The density function $f_{(t+1)}(m_{\rho(t+1)} \backslash \rho_{(t)}) = N(\mu_{\rho(t+1)}, v_{\rho(t+1)}^2)$ is no longer a probability function ($v^2_{\rho(t+1)} \to 0$) but a single real number $\mu_{\rho(\infty)}$ at time $t \to \infty$. The retailer, who can not revise his prior probability density function any further, will quote $P_{(\infty)}$ (see Eq. (2.5)) as his optimal market price, where

$$P_{(\infty)} = k_2(1+\varepsilon)^{-1}(\varepsilon + \mu_{\rho(\infty)} + V_\rho^2 \ln P_{(\infty)}) \\ \exp(\mu_{\rho(\infty)} \ln P_{(\infty)} + .5V_\rho^2(\ln P_{(\infty)})^2). \qquad (3.2)$$

The value of $P_{(\infty)}$ depends on the mean $\mu_{\rho(\infty)}$ and the variance V_ρ^2 assuming the market demand elasticity ε and k_2 are given.

If $P_{(\infty)} > P_{R(\infty)}$ at time $t \to \infty$, then we define $(P_{R(\infty)}, P_{(\infty)})$ as the limiting equilibrium point in this multistage bargaining game.

The manufacturer's profit $\pi_{R(\infty)}$ at time $t \to \infty$ is

$$\begin{aligned}\pi_{R(\infty)} &= (P_{R(\infty)} - c)q_{(\infty)} \\ &= (P_{R(\infty)} - c)(aP^{\varepsilon}_{(\infty)}).\end{aligned}$$

The retailer's profit $\pi_{(\infty)}$ at time $t \to \infty$ is

$$\begin{aligned}\pi_{(\infty)} &= (P_{(\infty)} - P_{R(\infty)})q_{(\infty)} \\ &= (P_{(\infty)} - P_{R(\infty)})(aP^{\varepsilon}_{(\infty)}).\end{aligned}$$

Though theoretically there exists a limiting equilibrium point $(P_{R(\infty)}, P_{(\infty)})$ in this model, the computer output fails to generate this equilibrium point. Before discussing how P_R and P change in the pricing process, we have to examine how the manufacturer's optimal retail price P_R and the retailer's optimal market price P are affected by factors of $(c, \varepsilon, \mu_\theta, \sigma_\theta^2)$ and $(k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$ in this model. The properties of the manufacturer's optimal retail price P_R and the retailer's optimal market price are the basis for understanding the simulation process in this model.

Properties of Manufacturer's Optimal Retail Price

From the manufacturer's objective function

$$\underset{P_R}{\mathrm{Max}}\ E_\theta[\pi_M] = ak_1^{\varepsilon}(P_R - c)\exp(\varepsilon\mu_\theta \ln P_R + .5\varepsilon^2\sigma_\theta^{\,2}(\ln P_R)^2)$$

and the objective function's first-order condition

$$P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^{\,2}\ln P_R(P_R - c) = 0,$$

the optimal retail price function $F(P_R|c, \varepsilon, \mu_\theta, \sigma_\theta^{\,2})$ is defined as

$$\begin{aligned} &F(P_R|c, \varepsilon, \mu_\theta, \sigma_\theta^{\,2}) \\ &\equiv P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^{\,2}\ln P_R(P_R - c) = 0. \end{aligned} \tag{3.3}$$

From Eq. (3.3), we notice the four factors, c, ε, μ_θ, and $\sigma_\theta^{\,2}$, will impact the manufacturer's optimal retail price P_R. The coefficient k_1 of $P = k_1P_R^{\theta}$ does not appear in Eq. (3.3), but k_1 does affect the manufacturer's optimal strategies. At time t, the observation $\theta_{(t)}$ (defined as $\theta_{(t)} = \ln(P_{(t)}/k_1)/\ln P_{R(t)}$) is used to revise $\mu_{\theta(t)}$ and $\sigma^2_{\theta(t)}$ to obtain the $\mu_{\theta(t+1)}$ and $\sigma^2_{\theta(t+1)}$, and the revised values of $\mu_{\theta(t+1)}$ and $\sigma^2_{\theta(t+1)}$ will affect optimal retail price P_R at time t+1. So the coefficient k_1 of $P = k_1P_R^{\theta}$ does affect the manufacturer's optimal strategy even it is not in Eq. (3.3). If we assume k

= 1, then k_1 will have no impact to the manufacturer's optimal retail price P_R.

Though $P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0$ is not an analytical function, we can show that the signs of $\partial P_R/\partial\mu_\theta$, $\partial P_R/\partial\sigma_\theta^2$, $\partial P_R/\partial\varepsilon$, and $\partial P_R/\partial c$ are robust over the feasible spectrum of the relevant parameters. The relationship between the optimal retail price P_R and $(c, \varepsilon, \mu_\theta, \sigma_\theta^2)$ is stated below from Property 1M to Property 4M.

Property 1M: If the variance σ_θ^2 increases, the manufacturer will quote a higher optimal retail price P_R. If the variance σ_θ^2 decreases, the manufacturer will quote a lower optimal retail price P_R.

Proof:
From $P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0$, we have

$$(\partial P_R/\partial\sigma_\theta^2) + \varepsilon\mu_\theta(\partial P_R/\partial\sigma_\theta^2) + \varepsilon^2\sigma_\theta^2 \ln P_R(\partial P_R/\partial\sigma_\theta^2) + \varepsilon^2(P_R - c)\ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)(\partial P_R/\partial\sigma_\theta^2) = 0.$$

$$(\partial P_R/\partial\sigma_\theta^2)\{1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)\} + \varepsilon^2(P_R - c)\ln P_R = 0.$$

Because $\varepsilon^2(P_R - c)\ln P_R > 0$, if

$$1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c) < 0, \tag{3.4}$$

we must have $\partial P_R/\partial \sigma_\theta^2 > 0$. With $\partial P_R/\partial \sigma_\theta^2 > 0$, the Property 1M holds. Equation (3.4) can be simplified as

$$P_R(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2) - c\varepsilon^2\sigma_\theta^2 < 0.$$

Because $c\varepsilon^2\sigma_\theta^2 > 0$ and $P_R > 0$, as long as

$$(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2) < 0,$$

$\partial P_R/\partial \sigma_\theta^2 > 0$ will always hold. In this dissertation the condition $(1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2) < 0$ will hold by assigning proper values to ε, μ_θ and σ_θ^2.

Property 1M implies if the manufacturer is more uncertain (a higher σ_θ^2) about the retailer's price-reaction, then he will intend to quote a higher retail price P_R. If the manufacturer is less uncertain (a lower σ_θ^2) about the retailer's price-reaction, then he will intend to quote a lower retail price P_R.

The computer-generated numerical output about the relationship between the variance σ_θ^2 and the manufacturer's optimal retail price P_R is listed in Appendix 4. In Appendix 4, the "******" symbol means the value is too big (overflow). The numerical values in Appendix 4 provide us good references to select the reasonable range of the manufacturer's optimal retail price P_R with the proper initial values of μ_θ and σ_θ^2 (c and ε are given) to start the simulation. One sample of the data is listed in Table 3-1 and plotted in Figure 3-1.

Table 3-1. Relationship between the manufacturer's optimal retail price P_R and the variance σ_θ^2.

μ_θ =	c =	ε =
1.6000	1.0000	-1.1000

σ_θ^2 =	P_R =
0.013	2.34
0.019	2.35
0.025	2.36
0.031	2.37
0.037	2.39
0.043	2.40
0.049	2.41
0.055	2.43
0.061	2.44
0.067	2.45
0.073	2.47
0.079	2.49
0.085	2.50
0.091	2.52

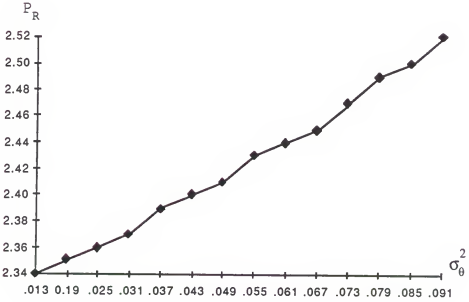

Figure 3-1. Relationship between the manufacturer's optimal retail price P_R and the variance σ_θ^2 with data from Table 3-1.

Property 2M: If the mean μ_θ goes up, the manufacturer will quote a lower optimal retail price P_R. If the mean μ_θ goes down, the manufacturer will quote a higher optimal retail price P_R.

Proof:

From $P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0$, we have

$$(\partial P_R/\partial\mu_\theta) + \varepsilon(P_R - c) + \varepsilon\mu_\theta(\partial P_R/\partial\mu_\theta) + \varepsilon^2\sigma_\theta^2 \ln P_R(\partial P_R/\partial\mu_\theta) + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)(\partial P_R/\partial\mu_\theta) = 0.$$

$$(\partial P_R/\partial\mu_\theta)\{1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)\} + \varepsilon(P_R - c) = 0.$$

Because $\varepsilon(P_R - c) < 0$, if

$$1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c) < 0, \qquad (3.4)$$

we must have $\partial P_R/\partial\mu_\theta < 0$. With $\partial P_R/\partial\mu_\theta < 0$, the Property 2M holds.

The computer-generated numerical output about the relationship between the mean μ_θ and the manufacturer's optimal retail price P_R is also listed in Appendix 4. One sample of the data is listed in Table 3-2 and plotted in Figure 3-2. Comparing Figure 3-1 and Figure 3-2, we notice that the impact of the change of the mean μ_θ with respect to

Table 3-2. Relationship between the manufacturer's optimal retail price P_R and the mean μ_θ.

$\sigma_\theta^2 =$	$c =$	$\varepsilon =$
0.0300	1.0000	-1.1000

$\mu_\theta =$	$P_R =$
1.20	4.80
1.25	4.09
1.30	3.61
1.35	3.26
1.40	3.00
1.45	2.79
1.50	2.63
1.55	2.49
1.60	2.37
1.65	2.27
1.70	2.19
1.75	2.11
1.80	2.05
1.85	1.99
1.90	1.94

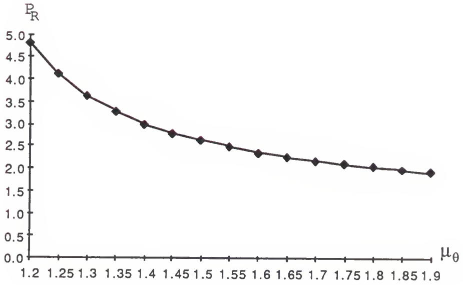

Figure 3-2. Relationship between the manufacturer's optimal retail price P_R and the mean μ_θ with data from Table 3-2.

the optimal retail price P_R is more significant than the impact of the change of the variance σ_θ^2 with respect to the optimal retail price P_R.

Property 3M: If the absolute value of the market demand elasticity ε goes up, the manufacturer will quote a lower optimal retail price P_R. If the absolute value of the market demand elasticity ε goes down, the manufacturer will quote a higher retail price P_R.

Proof:
From $P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0$, we have

$$(\partial P_R/\partial\varepsilon) + \mu_\theta(P_R - c) + \varepsilon\mu_\theta(\partial P_R/\partial\varepsilon) + 2\varepsilon\sigma_\theta^2(P_R - c)\ln P_R + \varepsilon^2\sigma_\theta^2(\partial P_R/\partial\varepsilon)\ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)(\partial P_R/\partial\varepsilon) = 0.$$

$$(\partial P_R/\partial\varepsilon)\{1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2\ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)\} + (P_R - c)(\mu_\theta + 2\varepsilon\sigma_\theta^2\ln P_R) = 0.$$

Assuming $(\mu_\theta + 2\varepsilon\sigma_\theta^2\ln P_R) > 0$, if

$$1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2\ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c) < 0, \qquad (3.4)$$

we have $\partial P_R/\partial\varepsilon > 0$. With $\partial P_R/\partial\varepsilon > 0$ ($\varepsilon < 0$), the Property 3M holds.

Property 3M implies the manufacturer intends to quote a higher retail price P_R in a less elastic market. When the market demand is more elastic, the manufacturer intends to quote a lower retail price P_R.

The computer-generated numerical output about the relationship between the market demand elasticity ε and the manufacturer's optimal retail price P_R is listed in Appendix 4. One sample of the data is listed in Table 3-3 and plotted in Figure 3-3.

Property 4M: If the unit cost c goes up, the manufacturer will quote a higher optimal retail price. If the unit cost c goes down, the manufacturer will quote a lower optimal retail price.

Proof:

From $P_R + \varepsilon\mu_\theta(P_R - c) + \varepsilon^2\sigma_\theta^2 \ln P_R(P_R - c) = 0$, we have

$$(\partial P_R/\partial c) + \varepsilon\mu_\theta(\partial P_R/\partial c - 1) + \varepsilon^2\sigma_\theta^2 \ln P_R(\partial P_R/\partial c - 1) + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)(\partial P_R/\partial c) = 0.$$

$$(\partial P_R/\partial c)\{1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c)\} - (\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R) = 0.$$

Because $\varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R < 0$, if

$$1 + \varepsilon\mu_\theta + \varepsilon^2\sigma_\theta^2 \ln P_R + \varepsilon^2\sigma_\theta^2(1/P_R)(P_R - c) < 0, \qquad (3.4)$$

we have $\partial P_R/\partial c > 0$. With $\partial P_R/\partial c > 0$, the Property 4M holds.

Table 3-3. Relationship between the manufacturer's optimal retail price P_R and the demand elasticity ε.

$\mu_\theta =$	$\sigma_\theta{}^2 =$	$c =$
1.8000	0.0500	1.0000

$\varepsilon =$	$P_R =$
-1.02	2.26
-1.04	2.21
-1.06	2.16
-1.08	2.11
-1.10	2.07
-1.12	2.03
-1.14	1.99
-1.16	1.96
-1.18	1.93
-1.20	1.90
-1.22	1.87
-1.24	1.84
-1.26	1.82
-1.28	1.80
-1.30	1.77

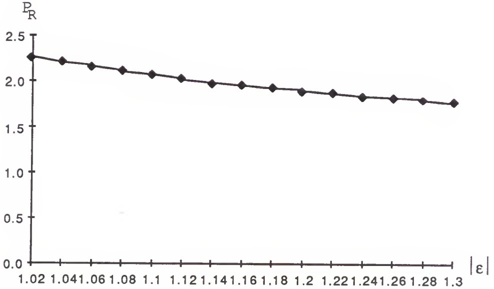

Figure 3-3. Relationship between the manufacturer's optimal retail price P_R and the demand elasticity $|\varepsilon|$ with data from Table 3-3.

Property 4M implies the manufacturer will raise the retail price P_R if the unit cost c increases. This relationship is clearer with $\sigma_\theta^2 = 0$. Under certainty ($\sigma_\theta^2 = 0$), the optimal retail price P_R becomes

$$P_R = c\left(\frac{\varepsilon\mu_\theta}{1 + \varepsilon\mu_\theta}\right).$$

We notice there is a linear relationship between P_R and c when ε and μ_θ are given. This implies the manufacturer intends to charge the retailer with a fixed percentage of the unit cost c.

The computer-generated numerical output about the relationship between the unit cost c and P_R is listed in Appendix 4. One sample of the data is listed in Table 3-4 and plotted in Figure 3-4. From Figure 3-4, we notice there exists a near·linear relationship between the unit cost c and the manufacturer's optimal retail price P_R.

Property 1M to property 4M are summarized as below (P_R^* is the optimal retail price P_R).

1M)	σ_θ^2	$\Uparrow$	$\Rightarrow$	P_R^*	$\Uparrow$
2M)	μ_θ	$\Uparrow$	$\Rightarrow$	P_R^*	$\Downarrow$
3M)	$\lvert\varepsilon\rvert$	$\Uparrow$	$\Rightarrow$	P_R^*	$\Downarrow$
4M)	c	$\Uparrow$	$\Rightarrow$	P_R^*	$\Uparrow$

Table 3-4. Relationship between the manufacturer's optimal retail price P_R and the unit cost c .

$\mu_\theta =$	$\sigma_\theta^2 =$	$\varepsilon =$
1.6000	0.0500	-1.1000

c =	P_R =
1.00	2.42
1.10	2.67
1.20	2.93
1.30	3.18
1.40	3.44
1.50	3.70
1.60	3.96
1.70	4.23
1.80	4.49
1.90	4.75
2.00	5.02
2.10	5.29
2.20	5.55
2.30	5.82
2.40	6.09

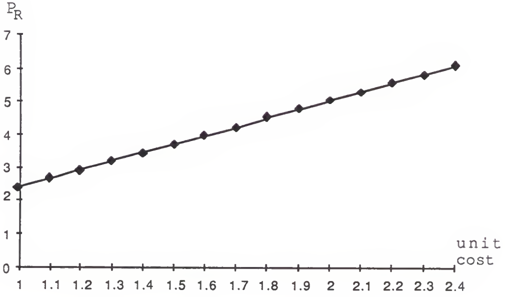

Figure 3-4. Relationship between the manufacturer's optimal retail price P_R and the unit cost c with data from Table 3-4.

Properties of Retailer's Optimal Market Price

From the retailer's objective function

$$\underset{P}{\text{Max}}\ E_\rho[\pi_R] = aP^{1+\varepsilon} - ak_2P^{\varepsilon}\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$$

and the first-order condition

$$(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2) = 0,$$

we define the retailer's optimal market price function $G(P|k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$ as

$$\begin{aligned} &G(P|k_2, \varepsilon, \mu_\rho, \sigma_\rho^2) \\ &\equiv (1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2) \\ &= 0. \end{aligned} \tag{3.5}$$

The manufacturer's unit cost c is not in the retailer's optimal market price function $G(P|k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$. This is an interesting property of the retailer's optimal market price P in this model. We may say the effect of the manufacturer's unit cost c is implicitly reflected by the subjective probability function $f(\rho) = N(\mu_\rho, \sigma_\rho^2)$ assigned to the price-reaction elasticity ρ. Because this special property, the numerical relationship between the retailer's optimal market price P and $(k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$, is especially important in selecting the proper initial values of $(k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$ to simulate the model.

Though $(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$ $= 0$ is not an analytical function, we can show that the signs of $\partial P/\partial\mu_\rho$, $\partial P/\partial\sigma_\rho^2$, $\partial P/\partial\varepsilon$, and $\partial P/\partial k_2$ are robust over the feasible spectrum of the relevant parameters by properly assigning values to ε, μ_ρ, and σ_ρ^2.

The relationship between the retailer's optimal market price P and the four factors $(k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$ is stated from Property 1R to Property 4R.

> Property 1R: If the variance σ_ρ^2 goes up, the retailer will quote a lower optimal market price P. If the variance σ_ρ^2 goes down, the retailer will quote a higher optimal market price P.

Proof:

From $(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2) =$ 0 and letting $\exp[\ ] \equiv \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$, we have

$$(1+\varepsilon)(\partial P/\partial\sigma_\rho^2) - k_2(\ln P + \sigma_\rho^2(\partial P/\partial\sigma_\rho^2)/P)\exp[\ ] - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp[\ ]\{\mu_\rho(\partial P/\partial\sigma_\rho^2)/P + .5(\ln P)^2 + \sigma_\rho^2(\ln P)(\partial P/\partial\sigma_\rho^2)/P\} = 0.$$

By substituting $(1+\varepsilon)P = k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp[\ ]$, we have

$$(\partial P/\partial\sigma_\rho^2)\{(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2 \ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ]\} - (1 + .5\ln P)k_2 \ln P \exp[\ ] = 0.$$

Because $(1 + .5\ln P)k_2\ln P\exp[\;] > 0$, if

$$(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2\ln P) - k_2\sigma_\rho^2(1/P)\exp[\;] < 0, \qquad (3.6)$$

we must have $\partial P/\partial\sigma_\rho^2 < 0$. In Eq (3.6), $k_2\sigma_\rho^2(1/P)\exp[\;] > 0$ and $(1+\varepsilon) < 0$, therefore as long as $(1 - \mu_\rho - \sigma_\rho^2\ln P) > 0$, Eq. (3.6) will always hold. In this dissertation the condition $(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2\ln P) - k_2\sigma_\rho^2(1/P)\exp[\;] < 0$ will hold by assigning proper values to ε, μ_ρ and σ_ρ^2.

The computer-generated numerical output about the relationship between the retailer's optimal market price P and the variance σ_ρ^2 is listed in Appendix 5. One sample of the numerical relationship between the retailer's optimal market price P and σ_ρ^2 is listed in Table 3-5 and plotted in Figure 3-5.

Property 2R: If the mean μ_ρ goes up, the retailer will quote a lower optimal market price P. If the mean μ_ρ goes down, the retailer will quote a higher optimal market price P.

Proof:

From $(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp(\mu_\rho\ln P + .5\sigma_\rho^2(\ln P)^2) = 0$ and letting $\exp[\;] \equiv \exp(\mu_\rho\ln P + .5\sigma_\rho^2(\ln P)^2)$, we have

$$(1+\varepsilon)(\partial P/\partial\mu_\rho) - k_2(1 + \sigma_\rho^2(\partial P/\partial\mu_\rho)/P)\exp[\;] - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\;]\{\ln P + \mu_\rho(\partial P/\partial\mu_\rho)/P + \sigma_\rho^2(\ln P)(\partial P/\partial\mu_\rho)/P\} = 0.$$

Table 3-5. Relationship between the retailer's optimal market price P and the variance σ_ρ^2.

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.8000	-1.1000	1.0000

$\sigma_\rho^2 =$	P =
0.110	5.15
0.120	4.56
0.130	4.10
0.140	3.74
0.150	3.45
0.160	3.21
0.170	3.01
0.180	2.84
0.190	2.70
0.200	2.58
0.210	2.47
0.220	2.37
0.230	2.29
0.240	2.22
0.250	2.15

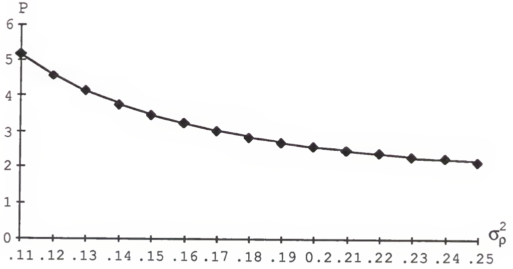

Figure 3-5. Relationship between the retailer's optimal market price P and the variance σ_ρ^2 with data from Table 3-5.

$$(\partial P/\partial\mu_\rho)\{(1+\varepsilon) - k_2\sigma_\rho^2(1/P)\exp[\ ] - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\ ]$$
$$[\mu_\rho(\partial P/\partial\mu_\rho)/P + \sigma_\rho^2(\ln P)(\partial P/\partial\mu_\rho)/P]\} - k_2\exp[\ ]$$
$$- k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\ ]\ln P = 0.$$

With $(1+\varepsilon)P = k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\ ]$, we have

$$(\partial P/\partial\mu_\rho)\{(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2\ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ]\}$$
$$- [k_2\exp[\ ] + (1+\varepsilon)P\ln P] = 0.$$

Assuming $k_2\exp[\ ] + (1+\varepsilon)P\ln P > 0$, then from

$$(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2\ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ] < 0, \qquad (3.6)$$

we have $\partial P/\partial\mu_\rho < 0$. With $\partial P/\partial\mu_\rho < 0$, the Property 2R holds in this model.

The computer-generated numerical output about the relationship between the retailer's optimal market price P and the mean μ_ρ is listed in Appendix 5. One sample of the numerical relationship between the retailer's optimal market price P and the mean μ_ρ is listed in Table 3-6 and plotted in Figure 3-6. Comparing the data in Figure 3-5 and Figure 3-6, we notice that the impact of the change of the mean μ_ρ with respect to the optimal market price P is more significant than the impact of the change of the variance σ_ρ^2 with respect to the optimal market price P.

Table 3-6. Relationship between the retailer's optimal market price P and the mean μ_ρ.

$\sigma_\rho^2 =$	$\varepsilon =$	$k_2 =$
0.1500	-1.1000	1.0000

$\mu_\rho =$	P =
0.60	9.14
0.62	8.45
0.64	7.75
0.66	7.09
0.68	6.47
0.70	5.87
0.72	5.31
0.74	4.79
0.76	4.30
0.78	3.85
0.80	3.45
0.82	3.07
0.84	2.73
0.86	2.43
0.88	2.15

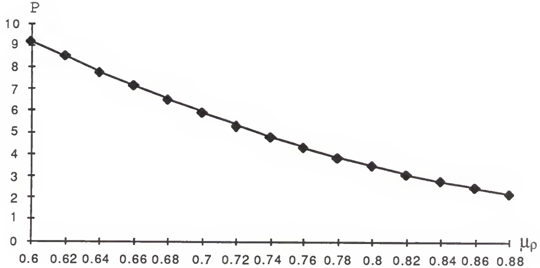

Figure 3-6. Relationship between the retailer's optimal market price P and the mean μ_ρ with data from Table 3-6.

Property 3R: If the absolute value of the market demand elasticity ε goes up, the retailer will quote a lower optimal market price P. If the absolute value of the market demand elasticity ε goes down, the retailer will quote a higher optimal market price P.

Proof:

From $(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2) = 0$ and letting $\exp[\ ] \equiv \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$, we have

$$P + (1+\varepsilon)(\partial P/\partial \varepsilon) - k_2(1 + \sigma_\rho^2(\partial P/\partial \varepsilon)/P)\exp[\ ] - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp[\ ]\{\mu_\rho(\partial P/\partial \sigma_\rho^2)/P + \sigma_\rho^2(\ln P)(\partial P/\partial \sigma_\rho^2)/P\} = 0.$$

With $(1+\varepsilon)P = k_2(\varepsilon + \mu_\rho + \sigma_\rho^2 \ln P)\exp[\ ]$, we have

$$(\partial P/\partial \varepsilon)\{(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2 \ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ]\} + [P - k_2\exp[\ ]] = 0.$$

If $P - k_2\exp[\ ] > 0$, then from

$$(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2 \ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ] < 0, \qquad (3.6)$$

we have $\partial P/\partial \varepsilon > 0$. With $\partial P/\partial \varepsilon > 0$ $(\varepsilon < 0)$, Property 3R implies the retailer will intend to quote a higher market price P in a less elastic market. When the market demand is more elastic, the retailer will intend to quote a lower market price P.

The computer-generated numerical output about the relationship between the retailer's optimal market price P and the market demand elasticity ε is listed in Appendix 5. One sample of the numerical relationship between the retailer's optimal market price P and the market demand elasticity ε is listed in Table 3-7 and plotted in Figure 3-7.

Though k_2 of $P_R = k_2P^\rho$ is in the retailer's optimal market price function $G(P|k_2, \varepsilon, \mu_\rho, \sigma_\rho^2)$, the price-reaction function $P_R = k_2P^\rho$ can be normalized with $k_2 = 1$ such that k_2 has no impact to the retailer's pricing strategies. As a reference, the relationship between the retailer's optimal market price P and k_2 is stated in Property 4R.

Property 4R: If k_2 goes up, the retailer will quote a higher optimal market price P. If k_2 goes down, the retailer will quote a lower optimal market price P.

Proof:

From $(1+\varepsilon)P - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2) = 0$ and letting $\exp[\;] \equiv \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2)$, we have

$$(1+\varepsilon)(\partial P/\partial k_2) - (\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\;] - k_2(\sigma_\rho^2(\partial P/\partial k_2)/P)\exp[\;] - k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P) \exp[\;]\{\mu_\rho(\partial P/\partial k_2)/P + \sigma_\rho^2(\ln P)(\partial P/\partial k_2)/P\} = 0.$$

With $(1+\varepsilon)P = k_2(\varepsilon + \mu_\rho + \sigma_\rho^2\ln P)\exp[\;]$, we obtain the simplified equation

Table 3-7. Relationship between the retailer's optimal market price P and the market demand elasticity ε.

$\mu_\rho =$	$\sigma_\rho^2 =$	$k_2 =$
0.8500	0.1200	1.0000

$\varepsilon =$	$P =$
-1.02	3.43
-1.04	3.38
-1.06	3.32
-1.08	3.27
-1.10	3.21
-1.12	3.16
-1.14	3.11
-1.16	3.06
-1.18	3.01
-1.20	2.97
-1.22	2.92
-1.24	2.88
-1.26	2.84
-1.28	2.79
-1.30	2.75

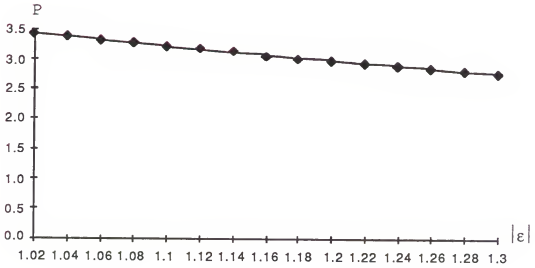

Figure 3-7. Relationship between the retailer's optimal market price P and the market demand elasticity $|\varepsilon|$ with data from Table 3-7.

$$(\partial P/\partial k_2)\{(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2 \ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ]\}$$
$$- (1+\varepsilon)P/k_2 = 0.$$

Because $(1+\varepsilon)P/k_2 < 0$, from

$$(1+\varepsilon)(1 - \mu_\rho - \sigma_\rho^2 \ln P) - k_2\sigma_\rho^2(1/P)\exp[\ ] < 0, \qquad (3.6)$$

we have $\partial P/\partial k_2 > 0$.

One sample of the computer-generated numerical relationship between the retailer's optimal market price P and k_2 is listed in Table 3-8 and plotted in Figure 3-8. From Figure 3-8, we notice the near linear relationship between the retailer's optimal market price P and k_2.

Property 1R to property 4R are summarized as below (P* is the optimal market price P).

1R) $\sigma_\rho^2 \Uparrow \quad \Rightarrow \quad P^* \Downarrow$
2R) $\mu_\theta \Uparrow \quad \Rightarrow \quad P^* \Downarrow$
3R) $|\varepsilon| \Uparrow \quad \Rightarrow \quad P^* \Downarrow$
4R) $k_2 \Uparrow \quad \Rightarrow \quad P^* \Uparrow$

Manufacturer's Pricing Strategies

At the end of time t, the manufacturer has two actions to take: he can either raise or lower his previous retail price P_R at the next pricing stage t+1. From

Table 3-8. Relationship between the retailer's optimal market price P and k_2.

$\mu_\rho =$	$\sigma_\rho^2 =$	$\varepsilon =$
0.8500	0.1200	-1.1000

k_2 =	P =
0.85	2.74
0.87	2.81
0.89	2.87
0.91	2.94
0.93	3.00
0.95	3.06
0.97	3.12
0.99	3.18
1.01	3.24
1.03	3.30
1.05	3.35
1.07	3.41
1.09	3.46
1.11	3.52
1.13	3.57

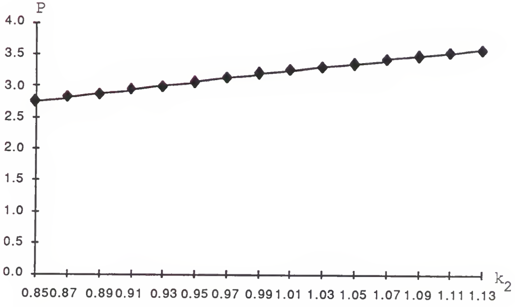

Figure 3-8. Relationship between the retailer's optimal market price P and k_2 with data from Table 3-8.

$$\theta_{(t)} = \ln(P_{(t)}/k_1)/\ln P_{R(t)},$$

$$\mu_{\theta(t+1)} = \frac{\theta_{(t)} v^2_{\theta(t)} + \mu_{\theta(t)} V^2_{\theta}}{v^2_{\theta(t)} + V^2_{\theta}},$$

$$v^2_{\theta(t+1)} = \frac{v^2_{\theta(t)} V^2_{\theta}}{v^2_{\theta(t)} + V^2_{\theta}} < v^2_{\theta(t)},$$

$$\sigma^2_{\theta(t+1)} = \frac{v^2_{\theta(t)} V^2_{\theta}}{v^2_{\theta(t)} + V^2_{\theta}} + V^2_{\theta},$$

we notice that a higher market price $P_{(t)}$ quoted by the retailer will cause a higher $\theta_{(t)}$, and a higher $\theta_{(t)}$ will cause a higher $\mu_{\theta_{(t+1)}}$. In general, we have the following situation:

If $\theta_{(t)} > \mu_{\theta_{(t)}}$, then $\mu_{\theta_{(t+1)}} > \mu_{\theta_{(t)}}$.
If $\theta_{(t)} < \mu_{\theta_{(t)}}$, then $\mu_{\theta_{(t+1)}} < \mu_{\theta_{(t)}}$.
If $\theta_{(t)} = \mu_{\theta_{(t)}}$, then $\mu_{\theta_{(t+1)}} = \mu_{\theta_{(t)}}$.

From Property 2M, we know a higher mean μ_θ will cause the manufacturer to quote a lower optimal retail price P_R, and a lower mean μ_θ will cause the manufacturer to quote a higher optimal retail price P_R. From Property 1M, we know a lower variance will cause the manufacturer to quote a lower optimal retail price P_R.

If $\theta_{(t)} > \mu_{\theta(t)}$, with $\mu_{\theta(t+1)} > \mu_{\theta(t)}$ and $v^2_{\theta(t+1)} < v^2_{\theta(t)}$, the manufacturer will quote a lower optimal retail price P_R at time t+1. If $\theta_{(t)} < \mu_{\theta(t)}$, with $\mu_{\theta(t+1)} < \mu_{\theta(t)}$ and $v^2_{\theta(t+1)} < v^2_{\theta(t)}$, the manufacturer's optimal retail price P_R is somewhat indeterminate because the mean μ_θ and the variance v_θ^2 have the opposite impact on the manufacturer's optimal retail price P_R. But from the numerical values in Appendix 4, we notice the mean μ_θ has greater impact on the manufacturer's optimal retail price P_R than the variance v_θ^2 does. So usually, if $\theta_{(t)} < \mu_{\theta_{(t)}}$, the manufacturer will quote a higher retail price P_R. If the special case of $\theta_{(t)} = \mu_{\theta_{(t)}}$ happens, then $P_{R(t+1)} < P_{R(t)}$. The manufacturer's pricing strategies can be stated as

> Property 5M: In general, at the end of time t, if the manufacturer observes a higher market price $P_{(t)}$ such that $\theta_{(t)} > \mu_{\theta_{(t)}}$, then he will quote a lower optimal retail price $P_{R(t+1)}$ at time t+1. If the manufacturer observes a lower market price $P_{(t)}$ such that $\theta_{(t)} < \mu_{\theta(t)}$, then he will quote a higher optimal retail price $P_{R(t+1)}$ at time t+1.

Property 5M can be summarized as

A higher $P_{(t)} \Rightarrow \theta_{(t)} > \mu_{\theta_{(t)}} \Rightarrow P_{R(t+1)} < P_{R(t)}$.
A lower $P_{(t)} \Rightarrow \theta_{(t)} < \mu_{\theta_{(t)}} \Rightarrow P_{R(t+1)} > P_{R(t)}$.

Property 5M implies that the manufacturer will raise the retail price P_R at time t+1, if he observes a "weaker" action (a lower market price P) taken by the retailer at time t. The manufacturer will lower the retail price P_R at time t+1, if he observes a "stronger" action (a higher market price P) taken by the retailer at time t. Under uncertainty, the manufacturer may implicitly have an expectation about the market price P after he has decided to quote P_R. We can define EP as the expected market price under P_R. For example, if the manufacturer decides to charge the retailer at $P_R = 2.1$ and guesses the retailer will quote the market price $P = 3.2$, then the market price $P = 3.2$ under $P_R = 2.1$ can be treated as the manufacturer's expected market price EP. If the manufacturer observes the market price P lower (or higher) than his expectation (the expected market price EP), then he will adjust the previous retail price P_R accordingly.

The expected market price EP can be used as an index to compare the observed market price P. From $\theta = \ln(P/k_1)/\ln P_R$, we substitute EP for P and define the expected observation of θ as $E\theta$, where

$$E\theta = \ln(EP/k_1)/\ln P_R.$$

Then EP and $E\theta$ can be used to infer the manufacturer's pricing strategies as if they were the manufacturer's actual expectation about the retailer's market price P. From $P = k_1 P_R^{\theta}$, we define EP and $E\theta$ as

EP : the manufacturer's expected market price

$$\begin{aligned} EP &\equiv E_\theta[P_{(t)}] \\ &= k_1 E_\theta[P_{R(t)}{}^\theta] \\ &= k_1 \exp(\mu_\theta \ln P_{R(t)} + .5\sigma_\theta{}^2(\ln P_{R(t)})^2). \end{aligned}$$

Eθ : the manufacturer's expected observation of θ

$$\begin{aligned} E\theta &\equiv \ln(EP/k_1)/\ln P_R \\ &= \ln(E[P]/k_1)/\ln P_R \\ &= \ln(E_\theta[P_R{}^\theta])/\ln P_R \\ &= \mu_\theta + .5\sigma_\theta{}^2 \ln P_R. \end{aligned}$$

Both EP and Eθ are determined by P_R because the manufacturer's expectation about P or θ is calculated after he has quoted the retail price P_R. If $\sigma_\theta{}^2$ is small, then we have $E\theta \rightarrow \mu_\theta$. So E$\theta$, like μ_θ in Property 5M, can be an index to adjust the manufacturer's pricing strategies. The Property 5M then is restated as the following:

> If the manufacturer observes the market price $P_{(t)}$ lower than the expected market price $EP_{(t)}$ at time t, then he will quote a higher retail price P_R at time t+1, where $P_{R(t+1)} > P_{R(t)}$. If the manufacturer observes the market price $P_{(t)}$ higher than the expected market price $EP_{(t)}$ at time t, then he will quote a lower retail price $P_{R(t+1)}$ at time t+1, where $P_{R(t+1)} < P_{R(t)}$.

If $P_{(t)} < EP_{(t)}$ then $P_{R(t+1)} > P_{R(t)}$.

If $P_{(t)} > EP_{(t)}$ then $P_{R(t+1)} < P_{R(t)}$.

The expected market price EP will be used to manipulate the initial pricing processes. For example, the manufacturer will lower his optimal retail price P_R at time t = 2, if P > EP at time t = 1. On the other hand, the manufacturer will raise his optimal retail price P_R at time t = 2, if P < EP at time t = 1. Though the actual optimal retail price P_R depends on both μ_θ and σ_θ^2 at time t, the expected market price EP makes it easy for us to manipulate the pricing processes in this model.

Retailer's Pricing Strategies

At the end of time t, the retailer has two actions to take that he either raises or lowers his previous market price P at time t+1. From

$$\rho_{(t)} = \ln(P_{R(t)}/k_2)/\ln P_{(t)},$$

$$\mu_{\rho(t+1)} = \frac{\rho_{(t)} v^2_{\rho(t)} + \mu_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}},$$

$$v^2_{\rho(t+1)} = \frac{v^2_{\rho(t)} V^2_{\rho}}{v^2_{\rho(t)} + V^2_{\rho}} < v^2_{\rho(t)},$$

$$\sigma^2_{\rho\,(t+1)} = \frac{v^2_{\rho\,(t)}\, V^2_{\rho}}{v^2_{\rho\,(t)} + V^2_{\rho}} + V^2_{\rho},$$

a higher retail price $P_{R(t)}$ quoted by the manufacturer will cause a higher $\rho_{(t)}$, and a higher $\rho_{(t)}$ will cause a higher $\mu_{\rho\,(t+1)}$. In general, we have the following situation:

If $\rho_{(t)} > \mu_{\rho\,(t)}$, then $\mu_{\rho\,(t+1)} > \mu_{\rho\,(t)}$.
If $\rho_{(t)} < \mu_{\rho\,(t)}$, then $\mu_{\rho\,(t+1)} < \mu_{\rho\,(t)}$.
If $\rho_{(t)} = \mu_{\rho\,(t)}$, then $\mu_{\rho\,(t+1)} = \mu_{\rho\,(t)}$.

From Property 2R, a higher mean μ_ρ will cause the retailer to quote a lower optimal market price P and a lower mean μ_ρ will cause the retailer to quote a higher optimal market price P. From Property 1R, a lower variance v_ρ^2 will cause the retailer to quote a higher optimal market price P. So if $\rho_{(t)} < \mu_{\rho(t)}$, with $\mu_{\rho(t+1)} < \mu_{\rho(t)}$ and $v^2_{\rho(t+1)} < v^2_{\rho(t)}$, the retailer will quote a higher optimal market price P at time t+1. If $\rho_{(t)} > \mu_{\rho(t)}$, with $\mu_{\rho(t+1)} < \mu_{\rho(t)}$ and $v^2_{\rho(t+1)} < v^2_{\rho(t)}$, the retailer's optimal market price P is somewhat indeterminate because the mean μ_ρ and the variance v_ρ^2 have the opposite impact to the retailer's optimal market price P. From the numerical values provided in Appendix 5, we notice the mean μ_ρ has greater impact to the retailer's optimal market price P than the variance v_ρ^2 does. So usually, if $\rho_{(t)} > \mu_{\rho(t)}$, the retailer will quote a lower market price P. If the special case of $\rho_{(t)} = \mu_{\rho_{(t)}}$ happens, then $P_{R(t+1)} > P_{R(t)}$. The retailer's pricing strategies are stated as:

> Property 5R: In general, at the end of time t, if the retailer observes a higher retail price $P_{R(t)}$ such that $\rho_{(t)} > \mu_{\rho_{(t)}}$, then he will quote a lower optimal market price $P_{(t+1)}$ at time t+1. If the retailer observes a lower retail price $P_{R(t)}$ such that $\rho_{(t)} < \mu_{\rho_{(t)}}$, then he will quote a higher market price $P_{(t+1)}$ at time t+1.

Property 5R can be summarized as:

A higher $P_{R(t)} \Rightarrow \rho_{(t)} > \mu_{\rho_{(t)}} \Rightarrow P_{(t+1)} < P_{(t)}$.

A lower $P_{R(t)} \Rightarrow \rho_{(t)} < \mu_{\rho_{(t)}} \Rightarrow P_{(t+1)} > P_{(t)}$.

Property 5R implies that the retailer will raise the market price P at time t+1 if he observes a "weaker" action (a lower retail price P_R) taken by the manufacturer at time t. The retailer will lower the market price P at time t+1 if he observes a "stronger" action (a higher retail price P_R) taken by the manufacturer at time t. Under uncertainty, the retailer may implicitly have an expectation about the retail price P_R after he has decided to quote P. We can define EP_R as the expected retail price under P. For example, after considering his bargaining position in this market, the retailer decides to quote the market price at P = 3.5 and guesses the manufacturer may quote the retail price at P_R =

2.8 (with c = 1). Now if the retailer observes the retail price P_R lower (higher) than his expectation (the expected retail price EP_R), then he will adjust the previous market price P accordingly.

The expected market price EP_R can be treated as an index to compare the observed retail price P_R. From

$$\rho = \ln(P_R/k_2)/\ln P,$$

we substitute the expected market price EP_R for P_R and define the expected observation of ρ as $E\rho$, where

EP_R : the retailer's expected retail price

$$\begin{aligned} EP_R &\equiv E_\rho[P_R] \\ &= k_2 E_\rho[P^\rho] \\ &= k_2 \exp(\mu_\rho \ln P + .5\sigma_\rho^2(\ln P)^2), \end{aligned}$$

$E\rho$: expected observation of ρ

$$\begin{aligned} E\rho &\equiv \ln(EP_R/k_2)/\ln P \\ &= \ln(E[P_R]/k_2)/\ln P \\ &= \ln(E_\rho[P^\rho])/\ln P \\ &= \mu_\rho + .5\sigma_\rho^2 \ln P. \end{aligned}$$

If the variance σ_ρ^2 is small (for a reasonable P), we can assume $E\rho \rightarrow \mu_\rho$. Then $E\rho$, like μ_ρ in Property 5R, can be used as the index to adjust the retailer's pricing strategies. The Property 5R can be restated as below:

If the retailer observes the retail price $P_{R(t)}$ lower than the expected retail price $EP_{R(t)}$ at time t, then he will quote a higher market price P at time t+1, where $P_{(t+1)} > P_{(t)}$. If the retailer observes the retail price $P_{R(t)}$ higher than the expected market price $EP_{R(t)}$ at time t, then he will quote a lower market price $P_{(t+1)}$ at time t+1, where $P_{(t+1)} < P_{(t)}$.

If $P_{R(t)} < EP_{R(t)}$ then $P_{(t+1)} > P_{(t)}$.
If $P_{R(t)} > EP_{R(t)}$ then $P_{(t+1)} < P_{(t)}$.

The expected retail price EP_R will be used to manipulate the initial pricing processes too. For example, the retailer will lower his optimal market price P at time t = 2 if we choose a manufacturer's optimal retail price P_R such that $P_R > EP_R$ at time t = 1. On the other hand, the retailer will raise his optimal market price P at time t = 2 if we choose a manufacturer's optimal retail price P_R such that $P_R < EP_R$ at time t = 1. Though the actual optimal market price P will depend on both μ_p and σ_p^2 at time t, the expected retail price EP_R is a good reference to manipulate the pricing processes in this model.

Pricing Patterns of This Model

From Property 5M and Property 5R, there exist four pricing patterns between time t and t+1 during the bargaining process. Assuming $P_{(t)} > P_{R(t)}$ and $P_{(t+1)} > P_{R(t+1)}$, the four pricing patterns are

Pattern 1: The manufacturer quotes a higher retail price P_R and the retailer quotes a lower market price P at time t+1.
$P_{R(t+1)} > P_{R(t)}$ because $\theta_{(t)} < \mu_{\theta(t)}$ or $P_{R(t)} < EP_{R(t)}$.
$P_{(t+1)} < P_{(t)}$ because $\rho_{(t)} > \mu_{\rho(t)}$ or $P_{R(t)} > EP_{R(t)}$.

Pattern 2: The manufacturer quotes a higher retail price P_R and the retailer quotes a higher market price P at time t+1.
$P_{R(t+1)} > P_{R(t)}$ because $\theta_{(t)} < \mu_{\theta(t)}$ or $P_{(t)} < EP_{(t)}$.
$P_{(t+1)} > P_{(t)}$ because $\rho_{(t)} < \mu_{\rho(t)}$ or $P_{R(t)} < EP_{R(t)}$.

Pattern 3: The manufacturer quotes a lower retail price P_R and the retailer quotes a lower market price P at time t+1.
$P_{R(t+1)} < P_{R(t)}$ because $\theta_{(t)} > \mu_{\theta(t)}$ or $P_{(t)} > EP_{(t)}$.
$P_{(t+1)} < P_{(t)}$ because $\rho_{(t)} > \mu_{\rho(t)}$ or $P_{R(t)} > EP_{R(t)}$.

Pattern 4: The manufacturer quotes a lower retail price P_R and the retailer quotes a higher market price P at time t+1.
$P_{R(t+1)} < P_{R(t)}$ because $\theta_{(t)} > \mu_{\theta(t)}$ or $P_{(t)} > EP_{(t)}$.
$P_{(t+1)} > P_{(t)}$ because $\rho_{(t)} < \mu_{\rho(t)}$ or $P_{R(t)} < EP_{R(t)}$.

Though at time t = 1 we can start with any one of these four pricing patterns, the bargaining process will eventually become a repeated Pattern 1 or Pattern 4 in this model. A repeated pricing Pattern 1 (the manufacturer keeps raising the retail price P_R and the retailer keeps lowering the market price P) will cause $P_R > P$ at some point of time. And a repeated pricing Pattern 4 (the manufacturer keeps on quoting a lower retail price P_R and the retailer keeps on quoting a higher market price P) will cause $P_R \rightarrow c$ and a very high market price P.

In the next chapter, we will discuss the pricing behavior in detail from the computer output generated by the multistage bargaining process to an examination of the equilibrium points in this model. The impact of the market demand elasticity ε on the final bargaining results will be discussed too.

CHAPTER IV
COMPUTER SIMULATED BARGAINING PROCESS

The Simulation Process

Before starting the simulation, we need to decide values of system factors in this model, where

$C = cq$	the manufacturer's total cost function,
$q = aP^{\varepsilon}$	the market demand function,
$P = k_1 P_R^{\theta}$	the manufacturer's price-reaction function,
$P_R = k_2 P^{\rho}$	the retailer's price-reaction function.
c	the manufacturer's unit cost,
ε	the market demand elasticity,
a	the coefficient of demand function,
k_1	the coefficient of $k_1 P_R^{\theta}$,
k_2	the coefficient of $k_2 P^{\rho}$,
q	the quantity purchases,
P_R	the manufacturer's retail price,
P	the retailer's market price,
θ	the manufacturer's price-reaction elasticity,
ρ	the retailer's price-reaction elasticity.

In this model, we set the unit cost $c = 1$, $\varepsilon = -1.1$, $k_1 = 1$, $k_2 = 1$, and $a = 10000$. The coefficient "a" in the market demand function, $q = aP^{\varepsilon}$, will not affect any player's pricing strategies, so its value is arbitrary. The market demand elasticity ε is set at $\varepsilon = -1.1$. Later we can change the value of ε to see its impact to the bargaining results. The six parameters $(\mu_\theta, v_\theta^2, V_\theta^2, \mu_\rho, v_\rho^2, V_\rho^2)$ are the initial input for the simulation model, where

μ_θ : the mean of $f(\theta) = N(\mu_\theta, \sigma_\theta^2)$,
V_θ^2 : the variance of $f(\theta \backslash m_\theta) = N(m_\theta, V_\theta^2)$,
v_θ^2 : the variance of $f(m_\theta) = N(\mu_\theta, v_\theta^2)$,

μ_ρ : the mean of $f(\rho) = N(\mu_\rho, \sigma_\rho^2)$,
V_ρ^2 : the variance of $f(\rho \backslash m_\rho) = N(m_\rho, V_\rho^2)$,
v_ρ^2 : the variance of $f(m_\rho) = N(\mu_\rho, v_\rho^2)$,

σ_θ^2 : the variance of $f(\theta) = N(\mu_\theta, \sigma_\theta^2)$,
σ_ρ^2 : the variance of $f(\rho) = N(\mu_\rho, \sigma_\rho^2)$,

$$\sigma_\theta^2 = v_\theta^2 + V_\theta^2,$$
$$\sigma_\rho^2 = v_\rho^2 + V_\rho^2.$$

Though we can start with any values of $(\mu_\theta, v_\theta^2, V_\theta^2, \mu_\rho, v_\rho^2, V_\rho^2)$ in this model, with the unit cost $c = 1$, the manufacturer's retail price P_R and the retailer's market price P must be in a reasonable range. For example, the cases of (1) $P_R = 4$ and $P = 4.5$, and (2) $P_R = 2$ and $P = 6.5$ will not be considered as the starting bargaining conditions. From Property 5M and Property 5R, the above starting bargaining conditions will cause (1) $P_R > P$, and (2) $P_R \rightarrow c$ and a very high P in a short period of time. What we are looking for is a reasonable price range such that the manufacturer and the retailer can bargain for a longer period of time.

From the analysis in Chapter III, there are four pricing patterns during the bargaining process.

Pattern 1: $P_R \Uparrow$ and $P \Downarrow$
Pattern 2: $P_R \Uparrow$ and $P \Uparrow$
Pattern 3: $P_R \Downarrow$ and $P \Downarrow$
Pattern 4: $P_R \Downarrow$ and $P \Uparrow$

We can start the bargaining process with any one of the four pricing patterns, but most processes will reach the pricing Pattern 1 and Pattern 4 very quickly and repeat the pricing Pattern 1 and Pattern 4 such that one of the following situation will happen:

Situation 1:

The manufacturer's retail price P_R is close to unit cost c, and the retailer's market price P is much greater than P_R.

Situation 2:

The manufacturer's retail price P_R is greater and greater and the retailer's market price P is smaller and smaller such that $P \rightarrow P_R$.

The limiting equilibrium point $(P_{R(\infty)}, P_{(\infty)})$ can not be reached in the simulation process because the variances, v^2_θ and v^2_ρ, are never equal to zero. When t → ∞, we have

$$v^2_\theta \to 0, \quad \sigma_\theta^2 \to V_\theta^2,$$
$$v^2_\rho \to 0, \quad \sigma_\rho^2 \to V_\rho^2,$$

$$P_{R(\infty)} = c(\varepsilon\mu_{\theta(\infty)} + \varepsilon^2 V_\theta^2 \ln P_{R(\infty)})(1 + \varepsilon\mu_{\theta(\infty)} + \varepsilon^2 V_\theta^2 \ln P_{R(\infty)})^{-1},$$

$$P_{(\infty)} = k_2(1+\varepsilon)^{-1}(\varepsilon + \mu_{\rho(\infty)} + V_\rho^2 \ln P_{(\infty)})$$
$$\exp(\mu_{\rho(\infty)} \ln P_{(\infty)} + .5V_\rho^2(\ln P_{(\infty)})^2).$$

In this model, most bargaining processes (with different values of μ_θ, σ_θ^2, μ_ρ and σ_ρ^2) are very short so that they have only 4 to 8 bargaining stages before reaching Situation 1 or Situation 2. A short bargaining process provides little information for us to infer the bargaining behavior and examine the equilibrium point. But there are a few bargaining processes which have longer pricing stages before reaching Situation 1 or Situation 2. In the next section, we analyze the simulation output which has 44 bargaining stages.

Simulation Output

The initial conditions of this bargaining process are listed below:

$a = 10000$
$c = 1.00$
$\varepsilon = -1.1$
$k_1 = 1.0$
$k_2 = 1.0$

$\mu_\theta = 1.65$
$v_\theta^2 = 0.01$
$V_\theta^2 = 0.03$
$\mu_\rho = 0.78$
$v_\rho^2 = 0.03$
$V_\rho^2 = 0.15$

We notice that the manufacturer's variance σ_θ^2 is very small, where

$$\begin{aligned}\sigma_\theta^2 &= v_\theta^2 + V_\theta^2\\ &= 0.01 + 0.03\\ &= 0.04.\end{aligned}$$

The manufacturer's σ_θ^2 variance must be small to obtain a reasonable optimal retail price P_R. The retailer's σ_ρ^2 is relatively greater than the manufacturer's variance σ_θ^2, where

$$\begin{aligned}\sigma_\rho^2 &= v_\rho^2 + V_\rho^2\\ &= 0.03 + 0.15\\ &= 0.18.\end{aligned}$$

The computer output is divided into four parts: (1) Figure 4-1, the optimal retail price P_R and the optimal market price P, (2) Figure 4-2 and Figure 4-3, the values of $\mu_{\theta(t)}$ and $\theta_{(t)}$, and the values of $\mu_{\rho(t)}$ and $\rho_{(t)}$, (3) Figure 4-4 and Figure 4-5, the variances of v_θ^2 and v_ρ^2, and (4) Figure 4-6 and Figure 4-7, the manufacturer's profit π_M and the retailer's profit π_R.

Table 4-1 and Figure 4-1

The manufacturer's retail price P_R and the expected market price EP, and the retailer's market price P and expected retail price EP_R are plotted in Figure 4-1 (the data are from Table 4-1). From Table 4-1 and Figure 4-1, we notice

(A) The manufacturer raises his retail price P_R from time t = 1 to t = 44. The retailer raises his market price P from time t = 1 to t = 25, then lowers his market price P from time t = 26 to t = 44. At time t = 44, $P_R > P$.

(B) The market price P is less than the expected market price EP from t = 1 to t = 44, so the manufacturer keeps increasing the retail price P_R (Property 5M). The retail price P_R is less than the expected price EP_R before t = 38, according to Property 5R, the retailer will keep increasing the market price P before t = 38. But the retailer starts to lower the market price at t = 26. As discussed in Chapter III, the expected market price EP or the expected retail price EP_R will not be a good index unless the variance σ_θ^2 or σ_ρ^2 is small.

The nonmonotonous pricing pattern (Figure 4-1) of the retail price P_R and market price P can be explained by the manufacturer and the retailer's pricing strategies (Property 5M and Property 5M).

Table 4-1. Simulation output of P_R , EP, P, and EP_R.

$a = 10000$ $c = 1.0$ $\varepsilon = -1.1$ $k_1 = 1.0$ $k_2 = 1.0$

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

t =	P_R =	EP =	P =	EP_R =
1	2.29	3.98	3.13	2.74
2	2.43	4.13	3.38	2.91
3	2.53	4.24	3.56	3.03
4	2.61	4.33	3.70	3.13
5	2.67	4.40	3.82	3.20
6	2.72	4.45	3.91	3.26
7	2.77	4.50	3.98	3.31
8	2.80	4.54	4.05	3.36
9	2.83	4.57	4.10	3.39
10	2.86	4.60	4.15	3.42
11	2.88	4.63	4.19	3.45
12	2.91	4.65	4.22	3.47
13	2.93	4.67	4.25	3.49
14	2.94	4.69	4.28	3.50
15	2.96	4.72	4.30	3.52
16	2.98	4.73	4.32	3.53
17	2.99	4.74	4.34	3.55
18	3.01	4.76	4.34	3.55
19	3.02	4.78	4.35	3.55
20	3.04	4.80	4.36	3.56
21	3.05	4.81	4.36	3.56
22	3.06	4.83	4.36	3.57
23	3.08	4.84	4.36	3.57
24	3.09	4.85	4.35	3.56
25	3.11	4.87	4.35	3.56
26	3.12	4.89	4.34	3.55
27	3.14	4.91	4.33	3.55
28	3.15	4.93	4.31	3.54
29	3.17	4.95	4.29	3.53
30	3.19	4.97	4.27	3.52
31	3.21	4.99	4.25	3.51
32	3.23	5.01	4.22	3.49
33	3.25	5.04	4.19	3.48
34	3.27	5.07	4.16	3.45
35	3.30	5.09	4.12	3.43
36	3.32	5.12	4.08	3.41
37	3.35	5.15	4.04	3.38
38	3.38	5.18	3.99	3.35
39	3.41	5.22	3.93	3.32
40	3.44	5.25	3.88	3.29
41	3.48	5.30	3.82	3.25
42	3.53	5.35	3.75	3.21
43	3.57	5.40	3.68	3.16
44	3.62	5.46	3.60	3.11
45	3.67	5.52	3.51	3.05

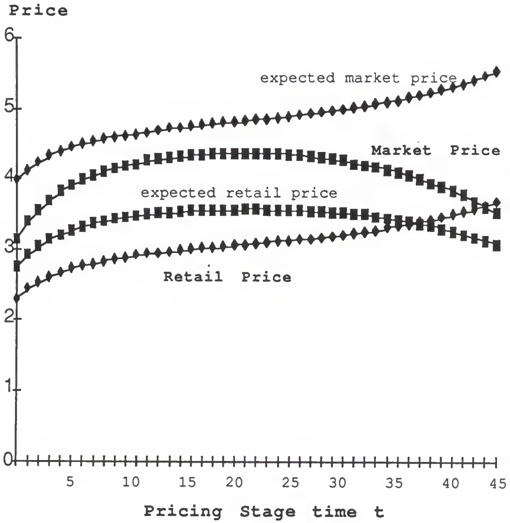

Figure 4-1. The manufacturer's retail price and the retailer's market price during the pricing process.

Property 5M:

A higher $P_{(t)} \Rightarrow \theta_{(t)} > \mu_{\theta_{(t)}} \Rightarrow P_{R(t+1)} < P_{R(t)}$.

A lower $P_{(t)} \Rightarrow \theta_{(t)} < \mu_{\theta_{(t)}} \Rightarrow P_{R(t+1)} > P_{R(t)}$.

Property 5R:

A higher $P_{R(t)} \Rightarrow \rho_{(t)} > \mu_{\rho_{(t)}} \Rightarrow P_{(t+1)} < P_{(t)}$.

A lower $P_{R(t)} \Rightarrow \rho_{(t)} < \mu_{\rho_{(t)}} \Rightarrow P_{(t+1)} > P_{(t)}$.

In fact, the nonmonotonous pricing pattern in Figure 4-1 is a quirk of the multistage bargaining model. The bargaining process starts with the pricing Pattern 2 at the beginning and ends up with the Pricing Pattern 1 at the later bargaining process.

Pattern 2: $P_R \Uparrow$ and $P \Uparrow$

Pattern 1: $P_R \Uparrow$ and $P \Downarrow$

The above pricing pattern will not be a defect in this model. If we consider the minimum profit as constraints in this model, then the bargaining process ends when these conditions are satisfied. For example, if the retailer only insists on a profit around \$2500, then he may stop bargaining at the moment he gets \$2500. Each player's variance of his prior probability distribution can also be the stopping rule. For instance, if the variance is lower than a certain value, then the bargaining process will be stopped.

Figure 4-2 and Figure 4-3

The values of $\mu_{\theta(t)}$ and $\theta_{(t)}$ during the pricing process are plotted in Figure 4-2 (the data are from Table 4-2), and the values of $\mu_{\rho(t)}$ and $\rho_{(t)}$ are plotted in Figure 4-3 (the data are from Table 4-3).

From Table 4-2 and Figure 4-2, we notice

(C) The value of $\theta_{(t)}$ (the observed price-reaction elasticity θ) is less than the mean $\mu_{\theta(t)}$ from time t = 1 to t = 44, so the manufacturer's retail price is increasing from time t = 1 to t = 44. In this case, both EP and $\theta_{(t)}$ are good indices.

From Table 4-3 and Figure 4-3, we notice

(D) The value of $\rho_{(t)}$ (the observed price-reaction elasticity ρ) is greater than the mean $\mu_{\rho(t)}$ after time t = 18. According to Property 5R, the retailer will lower his market price P after t = 18. But from Table 4-1, the retailer starts to lower the market price P at time t = 26.

Between time t = 18 and t = 26, the decreasing variance σ_ρ^2 has greater impact on increasing the market price P (Property 1R) than the increasing mean μ_ρ which causes a decreasing market price P (Property 2R).

Figure 4-4 and Figure 4-5

From Figure 4-4 (the data are list in Table 4-2), we notice

(E) The manufacturer's variance v_θ^2 = .002 at t = 13, and v_θ^2 = .0011 at t = 25. So the manufacturer's variance v_θ^2

Table 4-2. Simulation output of μ_θ, v_θ^2, σ_θ^2, and $\theta_{(t)}$.

$a = 10000$ $c = 1.0$ $\varepsilon = -1.1$ $k_1 = 1.0$ $k_2 = 1.0$

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

$t =$	$\mu_\theta =$	$v_\theta^2 =$	$\sigma_\theta^2 =$	$\theta_{(t)} =$
1	1.6500	0.0100	0.0400	1.3779
2	1.5820	0.0075	0.0375	1.3711
3	1.5398	0.0060	0.0360	1.3658
4	1.5108	0.0050	0.0350	1.3640
5	1.4898	0.0043	0.0343	1.3623
6	1.4739	0.0038	0.0338	1.3605
7	1.4613	0.0033	0.0333	1.3589
8	1.4510	0.0030	0.0330	1.3575
9	1.4425	0.0027	0.0327	1.3564
10	1.4354	0.0025	0.0325	1.3541
11	1.4291	0.0023	0.0323	1.3528
12	1.4237	0.0021	0.0321	1.3502
13	1.4188	0.0020	0.0320	1.3488
14	1.4144	0.0019	0.0319	1.3461
15	1.4104	0.0018	0.0318	1.3435
16	1.4067	0.0017	0.0317	1.3410
17	1.4032	0.0016	0.0316	1.3391
18	1.4000	0.0015	0.0315	1.3340
19	1.3969	0.0014	0.0314	1.3290
20	1.3938	0.0014	0.0314	1.3258
21	1.3908	0.0013	0.0313	1.3210
22	1.3879	0.0012	0.0313	1.3154
23	1.3850	0.0012	0.0312	1.3104
24	1.3821	0.0012	0.0312	1.3037
25	1.3792	0.0011	0.0311	1.2968
26	1.3763	0.0011	0.0311	1.2890
27	1.3733	0.0010	0.0310	1.2805
28	1.3702	0.0010	0.0310	1.2711
29	1.3670	0.0010	0.0310	1.2623
30	1.3637	0.0009	0.0309	1.2519
31	1.3603	0.0009	0.0309	1.2412
32	1.3568	0.0009	0.0309	1.2280
33	1.3531	0.0009	0.0309	1.2162
34	1.3493	0.0008	0.0308	1.2013
35	1.3453	0.0008	0.0308	1.1875
36	1.3412	0.0008	0.0308	1.1716
37	1.3368	0.0008	0.0308	1.1539
38	1.3323	0.0007	0.0307	1.1359
39	1.3275	0.0007	0.0307	1.1159
40	1.3224	0.0007	0.0307	1.0953
41	1.3172	0.0007	0.0307	1.0734
42	1.3116	0.0007	0.0307	1.0494
43	1.3058	0.0007	0.0307	1.0239
44	1.2997	0.0007	0.0307	0.9955
45	1.2932	0.0006	0.0306	0.9652

Table 4-3. Simulation output of μ_ρ, v_ρ^2, σ_ρ^2, and $\rho_{(t)}$.

$a = 10000$ $c = 1.0$ $\varepsilon = -1.1$ $k_1 = 1.0$ $k_2 = 1.0$

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

t =	$\mu_\rho =$	$v_\rho^2 =$	$\sigma_\rho^2 =$	$\rho_{(t)} =$
1	0.7800	0.0300	0.1800	0.7257
2	0.7710	0.0250	0.1750	0.7293
3	0.7650	0.0214	0.1714	0.7322
4	0.7609	0.0187	0.1688	0.7331
5	0.7578	0.0167	0.1667	0.7340
6	0.7554	0.0150	0.1650	0.7350
7	0.7536	0.0136	0.1636	0.7359
8	0.7521	0.0125	0.1625	0.7366
9	0.7509	0.0115	0.1615	0.7373
10	0.7499	0.0107	0.1607	0.7385
11	0.7492	0.0100	0.1600	0.7392
12	0.7486	0.0094	0.1594	0.7406
13	0.7481	0.0088	0.1588	0.7414
14	0.7477	0.0083	0.1583	0.7429
15	0.7475	0.0079	0.1579	0.7443
16	0.7473	0.0075	0.1575	0.7457
17	0.7472	0.0071	0.1571	0.7468
18	0.7472	0.0068	0.1568	0.7496
19	0.7473	0.0065	0.1565	0.7525
20	0.7475	0.0062	0.1563	0.7542
21	0.7478	0.0060	0.1560	0.7570
22	0.7482	0.0058	0.1558	0.7602
23	0.7486	0.0056	0.1556	0.7632
24	0.7491	0.0054	0.1554	0.7670
25	0.7497	0.0052	0.1552	0.7711
26	0.7505	0.0050	0.1550	0.7758
27	0.7513	0.0048	0.1548	0.7810
28	0.7522	0.0047	0.1547	0.7868
29	0.7532	0.0045	0.1545	0.7922
30	0.7544	0.0044	0.1544	0.7988
31	0.7557	0.0043	0.1543	0.8057
32	0.7570	0.0042	0.1542	0.8143
33	0.7586	0.0041	0.1541	0.8223
34	0.7603	0.0039	0.1539	0.8324
35	0.7621	0.0038	0.1538	0.8421
36	0.7641	0.0037	0.1538	0.8535
37	0.7663	0.0037	0.1537	0.8666
38	0.7687	0.0036	0.1536	0.8803
39	0.7713	0.0035	0.1535	0.8962
40	0.7741	0.0034	0.1534	0.9130
41	0.7772	0.0033	0.1533	0.9316
42	0.7806	0.0033	0.1533	0.9529
43	0.7842	0.0032	0.1532	0.9767
44	0.7882	0.0031	0.1531	1.0045
45	0.7927	0.0031	0.1531	1.0360

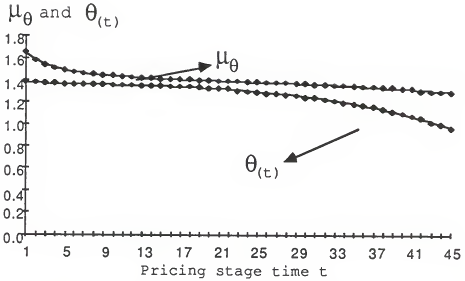

Figure 4-2. The values of the manufacturer's mean μ_θ and $\theta_{(t)}$ during the pricing process.

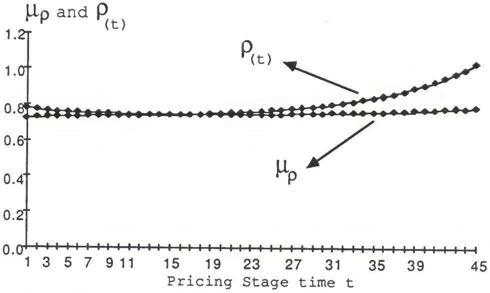

Figure 4-3. The values of the retailer's mean μ_ρ and $\rho_{(t)}$ during the pricing process.

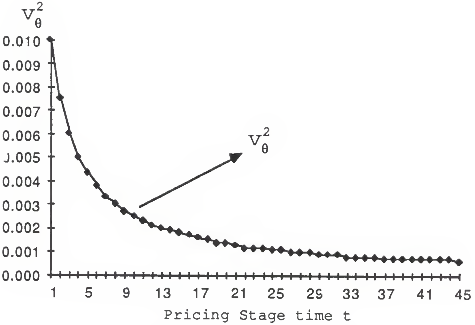

Figure 4-4. The manufacturer's decreasing variance v_θ^2 during the pricing process.

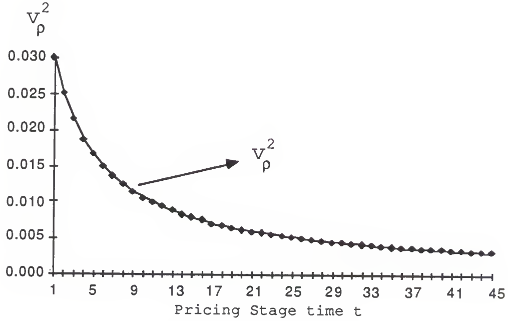

Figure 4-5. The retailer's decreasing variance v_ρ^2 during the pricing process.

drops to 20% and 10% of its starting value (v_θ^2 = .01 at t = 1) at t = 13 and t = 25 respectively.

From Figure 4-5 (the data are list in Table 4-3), we notice

(F) The variance $v_\rho^2 = .006$ at t = 20, and $v_\rho^2 = .0031$ at t = 44. So v_ρ^2 drops to 20% and 10% of its starting value (v_ρ^2 = .03 at time t = 1) at t = 20 and t = 44 respectively.

The variances, v_θ^2 and v_ρ^2, will never be equal to zero in this computer simulated bargaining process, so the limiting equilibrium point can't be reached in this model. But if v_θ^2 and v_ρ^2 are smaller than certain values, both players may treat them equal to zero such that the bargaining process ends.

Let us assume both players stop bargaining at time T. Then the manufacturer's variance is $\sigma_{\theta(T)}^2 = v_{\theta(T)}^2 + V_\theta^2$ and the retailer's variance is $\sigma_{\rho(T)}^2 = v_{\rho(T)}^2 + V_\rho^2$. With the values of $\mu_{\theta(T)}$ and $\mu_{\rho(T)}$ at time T, the equilibrium point $(P_{R(T)}, P_{R(T)})$ is

$$P_{R(T)} = c(\varepsilon\mu_{\theta(T)} + \varepsilon^2\sigma_{\theta(T)}^2 \ln P_{R(T)})(1 + \varepsilon\mu_{\theta(T)} + \varepsilon^2\sigma_{\theta(T)}^2 \ln P_{R(T)})^{-1},$$

$$P_{(T)} = k_2(1+\varepsilon)^{-1}(\varepsilon + \mu_{\rho(T)} + \sigma_{\rho(T)}^2 \ln P_{(T)})$$
$$\exp(\mu_{\rho(T)} \ln P_{(T)} + .5\sigma_{\rho(T)}^2(\ln P_{(T)})^2).$$

Figure 4-6 and Figure 4-7

From Figure 4-6 (data from Table 4-4), we notice

(G) The manufacturer's profit π_M increases from time t = 1 to t = 43. The retailer's profit π_R increases from t = 1 to

Table 4-4. Simulation output of q, π_M, π_R, and π_T.

$a = 10000$ $c = 1.0$ $\varepsilon = -1.1$ $k_1 = 1.0$ $k_2 = 1.0$

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

t =	q =	$\pi_M =$	$\pi_R =$	$\pi_T =$	$\pi_M/\pi_T =$	$\pi_R/\pi_T =$
1	2848.4	3674.4	2398.3	6072.7	0.60	0.40
2	2622.5	3747.3	2484.6	6231.9	0.60	0.40
3	2477.7	3793.9	2537.1	6331.0	0.59	0.40
4	2371.9	3816.7	2585.3	6402.0	0.59	0.40
5	2292.3	3833.6	2620.7	6454.4	0.59	0.40
6	2233.5	3847.9	2644.6	6492.5	0.59	0.40
7	2185.5	3859.4	2663.9	6523.3	0.59	0.40
8	2147.7	3868.6	2678.8	6547.3	0.59	0.41
9	2115.2	3876.1	2691.7	6567.8	0.59	0.41
10	2089.0	3888.5	2695.6	6584.1	0.59	0.41
11	2066.7	3895.4	2702.6	6598.0	0.59	0.41
12	2049.9	3908.6	2699.7	6608.4	0.59	0.40
13	2033.4	3916.3	2702.2	6618.5	0.59	0.40
14	2021.4	3929.3	2696.5	6625.8	0.59	0.40
15	2008.4	3942.7	2691.1	6633.8	0.59	0.40
16	2001.1	3955.1	2683.1	6638.2	0.59	0.40
17	1990.2	3964.5	2680.3	6644.8	0.59	0.40
18	1987.6	3989.7	2656.7	6646.4	0.60	0.40
19	1984.6	4014.7	2633.5	6648.2	0.60	0.39
20	1978.7	4030.7	2621.1	6651.8	0.60	0.39
21	1977.5	4055.3	2597.1	6652.5	0.60	0.39
22	1977.7	4083.7	2568.6	6652.4	0.61	0.38
23	1977.3	4110.2	2542.4	6652.6	0.61	0.38
24	1982.1	4144.9	2504.8	6649.7	0.62	0.37
25	1984.8	4181.5	2466.6	6648.1	0.62	0.37
26	1990.6	4223.8	2420.8	6644.6	0.63	0.36
27	1996.6	4270.5	2370.4	6640.9	0.64	0.35
28	2006.5	4322.8	2312.2	6634.9	0.65	0.34
29	2013.0	4372.9	2258.1	6630.9	0.65	0.34
30	2024.5	4432.8	2191.1	6624.0	0.66	0.33
31	2035.8	4496.1	2120.9	6617.0	0.67	0.32
32	2053.1	4575.7	2030.7	6606.4	0.69	0.30
33	2065.4	4649.5	1949.3	6598.8	0.70	0.29
34	2085.4	4744.2	1842.1	6586.4	0.72	0.28
35	2105.6	4834.8	1739.0	6573.8	0.73	0.26
36	2127.5	4942.4	1617.7	6560.0	0.75	0.24
37	2154.6	5065.8	1477.1	6542.9	0.77	0.22
38	2183.3	5196.2	1328.5	6524.7	0.79	0.20
39	2216.9	5347.2	1156.0	6503.2	0.88	0.11
40	2253.3	5508.5	971.3	6479.8	0.85	0.15
41	2290.5	5689.7	765.9	6455.6	0.88	0.11
42	2335.2	5897.4	528.9	6426.2	0.91	0.08
43	2386.5	6129.8	262.5	6392.2	0.96	0.04

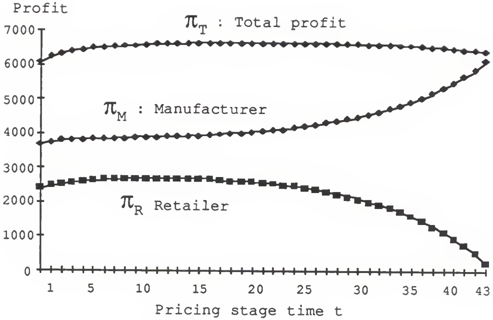

Figure 4-6. The profit share between the manufacturer's profit π_M and the retailer's profit π_R.

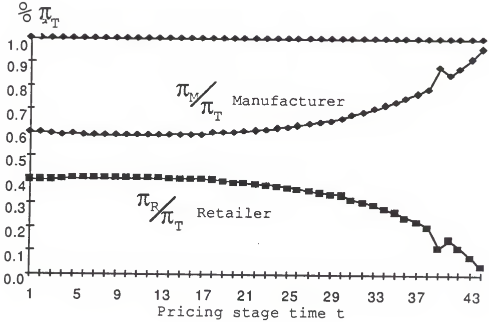

Figure 4-7. The percentage of total profit π_T between the manufacturer and the retailer.

t = 13 and decreases since t = 14. The total profit, $\pi_T = \pi_M + \pi_R$, increases from t = 1 to t = 23 and drops since t = 24. At time t = 44, $P_R > P$, the retailer's profit π_R is negative.

Because the bilateral trading is a non-zero-sum game, each player's relative bargaining position can be defined as the percentage of total profit, where

$\%\pi_M$: the manufacturer's share of the total profit π_T

$$\%\pi_M = \pi_M/(\pi_M+\pi_R),$$

$\%\pi_R$: the retailer's share of the total profit π_T

$$\%\pi_R = \pi_R/(\pi_M+\pi_R).$$

The manufacturer's $\%\pi_M$ and the retailer's $\%\pi_R$ are plotted in Figure 4-7 to examine the player's relative bargaining position. From Figure 4-7 (data from Table 4-4), we notice

(H) The manufacturer's $\%\pi_M$ is around 60% and the retailer's $\%\pi_R$ is around 40 % from time t =1 to t = 21. This is a very interesting result in this simulated bargaining process. From Figure 4-7, it seems the manufacturer and the retailer both insist on the 60% and 40% profit share during the bargaining process from t = 1 to t = 21. If both players satisfy the 60% and 40% profit share, then they may become cooperative in this market.

The computer output with the market demand elasticity $\varepsilon = -1.2$ is listed in Appendix 6, and the computer output with $\varepsilon = -1.5$ is listed in Appendix 7. From Appendix 6 and Appendix

7, we notice that (1) the bargaining process with a higher absolute value of the market demand elasticity ε reaches Situation 1 ($P > P_R$) much sooner, and (2) both P_R and P are lower if the absolute value of the demand elasticity ε is higher. This bargaining result is consistent with the manufacturer's pricing property (Property 3M) and the retailer's pricing property (Property 3R).

3M) $|\varepsilon| \Uparrow \quad \Rightarrow \quad P_R^* \Downarrow$

3R) $|\varepsilon| \Uparrow \quad \Rightarrow \quad P^* \Downarrow$

The computer output with the bargaining process that the manufacturer quotes the retail price P_R first is listed in Appendix 8. From Appendix 8, we notice that the market price P becomes much greater than the retail price P_R during the bargaining process (Situation 1). The bargaining result in Appendix 8 is opposite to the simultaneous bargaining process discussed in the previous section. The reason is at the present pricing rule (the manufacturer quotes the retail price P_R first), the retailer has observed P_R before he quotes P at time $t = 2$, because $P_R < EP_R$ at time $t = 1$, so a higher market price P is quoted at time $t = 2$ such that the bargaining result is reversed.

The computer output with the bargaining process that the retailer quotes the market price P first is listed in Appendix 9. From Appendix 9, the bargaining result becomes $P_R > P$ at time $t = 5$ (Situation 2). This bargaining process is much shorter than the simultaneous pricing process ($P_R > P$ at time $t = 44$). The reason is at the present pricing order (the retailer quotes the market price P first), the manufacturer has observed P before he quotes P_R at time $t = 1$, because $P < EP$ at time $t = 1$, so a much higher P_R at time $t = 2$ causes the bargaining process to reach Situation 2 in a much shorter period of time.

Though the bargaining results of different bargaining processes depend on the case-by-case basis, generally, we can infer that the player who quotes the price first may have the disadvantage in a dynamic pricing process. But in Samuelson's one-stage first-offer bargaining model (1980), the player who makes the first and final offer always confers a distinct bargaining advantage.

CHAPTER V
DISCUSSION AND FURTHER RESEARCH DIRECTIONS

Discussion

From the manufacturer and the retailer's pricing strategies (Property 5M and Property 5R), we realize that each player, the manufacturer and the retailer, acts somewhat like a price-adjustment machine in this bargaining process. Property 5M and Property 5R will make sense only under the assumptions that (1) both players will give the "true" subjective judgment about their bargaining positions to quote the retail price P_R and the market price P, and (2) they are willing to "compromise" according to the price signals (the observed P_R and P). Therefore cheating behavior is excluded in this model; otherwise, the retailer can quote a very high market price P such that the manufacturer will always lower the retail price P_R or the manufacturer will quote a very high retail price P_R such that the retailer will always lower the market price P.

The simulated Bayesian bargaining process has showed the following properties in this paper:

(1) The limiting equilibrium point can not be reached because the variance in each player's prior probability density function is never equal to zero in the computer program.

But by defining some threshold for v_θ^2 and v_ρ^2, say $v_{\theta(T)}^2$ and $v_{\rho(T)}^2$, and assuming the small $v_{\theta(T)}^2$ and $v_{\rho(T)}^2$ can not be detected by players, then the limiting equilibrium point exists in this model. When $v_{\theta(t)}^2 < v_{\theta(T)}^2$ and $v_{\rho(t)}^2 < v_{\rho(T)}^2$ at time t, the limiting equilibrium point $(P_{R(t)}, P_{(t)})$ becomes

$$P_{R(t)} = c(\varepsilon\mu_{\theta(t)} + \varepsilon^2\sigma_{\theta(t)}{}^2 \ln P_{R(t)})$$
$$(1 + \varepsilon\mu_{\theta(t)} + \varepsilon^2\sigma_{\theta(t)}{}^2 \ln P_{R(t)})^{-1},$$

$$P_{(t)} = k_2(1+\varepsilon)^{-1}(\varepsilon + \mu_{\rho(t)} + \sigma_{\rho(t)}{}^2 \ln P_{(t)})$$
$$\exp(\mu_{\rho(t)} \ln P_{(t)} + .5\sigma_{\rho(t)}{}^2(\ln P_{(t)})^2).$$

(2) By adding the minimum profit constraints in this model, the bargaining process will end when these conditions are satisfied. One interesting question is whether the two players will behave cooperatively in this market at the end of bargaining? Under certain circumstances, it seems that if information can be exchanged completely and each player's profit share is mutually acceptable, the bargaining game may end up with a vertical integrated market.

(3) The manufacturer and the retailer have opposite pricing strategies towards their variances σ_θ^2 and σ_ρ^2 in this model. When σ_θ^2 increases the manufacturer will raise the retail price P_R, while the retailer will lower the market price P when σ_ρ^2 increases. This may imply that if the manufacturer (as a seller) is more uncertain about the retailer's reactions, may prefer to quote a higher retail

price and let the retailer bargain the retail price down later. When the bargaining process proceeds, the variance σ_θ^2 becomes smaller, so the manufacturer will lower the retail price. The retailer, as a buyer, he may start at a lower market price P (the buying price) when he is more uncertain of the manufacturer's bargaining power. Later when the bargaining process proceeds, the variance σ_ρ^2 becomes smaller, so the retailer will raise the market price.

Further Research Directions

The further research directions in solving human bargaining problems are as following:

A) Because the connection between the empirical work and the game theory is somewhat less developed than might be hoped for, the simulated bargaining process can be the bridge connecting game theory and empirical work. The computer-simulated bargaining process can be compared to empirical gaming behavior.

B) How individuals update their information is an important issue in the multistage bargaining games. In this model the decision agents behave as "Bayesian" players to revise their prior subjective judgment. Different information revision methods may be applied to a bargaining process to examine their impact to the final equilibrium.

C) Players may take different strategies when they play a multistage bargaining game. In this model there are no game rules to stop the bargaining process. We may make some artificial game rules to examine these rules' impact on a multistage game.

D) The multistage game with the computer simulation, as seen in this paper, provides a new research direction in the study of the competitive behavior and market structure. If some game-situation is specified to a particular problem, then the bargaining model in this paper can help to examine the equilibrium points.

Although much work remains before human bargaining behavior may be formulated by a mathematical model, this paper provides a research direction in using the computer to simulate the bargaining processes.

APPENDIX 1
PROOF OF POSTERIOR DISTRIBUTIONS

Given $f(x|m) \sim N(m, \sigma_m^2)$ and $f(m) \sim N(\mu, \sigma^2)$, we prove $f(x) \sim N(\mu, \sigma_m^2 + \sigma^2)$.

Proof:

Letting $k_1 = \sqrt{(2\pi)}$ and $k = k_1^2 = 2\pi$, then

$$f(x) = \int f(x|m)\ f(m)\ dm$$

$$= \int (k_1\sigma_m)^{-1} \exp(-(x-m)^2/(2\sigma_m^2))$$
$$(k_1\sigma)^{-1} \exp(-(m-\mu)^2/(2\sigma^2))\ dm,$$

$$f(x) = (k\sigma_m\sigma)^{-1} \int \exp(-(\sigma^2(x^2-2mx+m^2)+\sigma_m^2(m^2-2m\mu+\mu^2))/(2\sigma_m^2\sigma^2))\ dm.$$

From $(\sigma^2(x^2-2mx+m^2)+\sigma_m^2(m^2-2m\mu+\mu^2))/(2\sigma_m^2\sigma^2)$

$$= (m-(x\sigma^2+\mu\sigma_m^2)/(\sigma_m^2+\sigma^2))^2/(2\sigma_m^2\sigma^2/(\sigma_m^2+\sigma^2))$$

$$+ (x-\mu)^2/(2(\sigma_m^2+\sigma^2)),$$

we have $f(x) = (\sqrt{k}(\sigma_m^2+\sigma^2))^{-1} \exp(-(x-\mu)^2/(2(\sigma_m^2+\sigma^2)))$

$$\{ \int (\sqrt{(\sigma_m^2+\sigma^2)}/\sqrt{(k\sigma_m^2\sigma^2)})^{-1}$$
$$\exp(-(m-(x\sigma^2+\mu\sigma_m^2)/(\sigma_m^2+\sigma^2))^2/(2\sigma_m^2\sigma^2/(\sigma_m^2+\sigma^2)))\ dm\}.$$

But $\{ \int (\sqrt{(\sigma_m^2+\sigma^2)}/\sqrt{(k\sigma_m^2\sigma^2)})^{-1}$
$\exp(-(m-(x\sigma^2+\mu\sigma_m^2)/(\sigma_m^2+\sigma^2))^2/(2\sigma_m^2\sigma^2/(\sigma_m^2+\sigma^2)))\ dm\}$
$= 1,$

so $f(x) = (\sqrt{2\pi}(\sigma_m^2+\sigma^2))^{-1} \exp(-(x-\mu)^2/(2(\sigma_m^2+\sigma^2))),$

we have $f(x) \sim N(\mu, \sigma_m^2+\sigma^2)$. The proof is complete.

APPENDIX 2

PROOF OF $E[P^x]$ AND $E[P^{\varepsilon x}]$

$x \sim N(\mu, \sigma^2)$ $N(\mu, \sigma^2)$ is a Normal distribution with mean μ and variance σ^2.

To prove

(1) $E[P^x] = \exp(\mu \ln P + .5\sigma^2(\ln P)^2)$,

(2) $E[P^{\varepsilon x}] = \exp(\varepsilon\mu \ln P + .5\varepsilon^2\sigma^2(\ln P)^2)$,

where $P > 1$, $\ln P > 0$, and ε is a constant.

Proof:

Letting $P = e^{\ln P}$, then

$E[P^x] = E[e^{(\ln P)x}]$.

From the Normal function's moment-generating function

$E[e^{tx}] = \exp(t\mu + .5t^2\sigma^2)$,

letting $t = \ln P$, we obtain

$$E[P^x] = \exp(\mu \ln P + .5\sigma^2(\ln P)^2). \qquad (1)$$

Letting $P = e^{\ln P}$, then

$E[P^{\varepsilon x}] = E[e^{\varepsilon(\ln P)x}]$.

letting $t = \varepsilon \ln P$, we obtain

$$E[P^{\varepsilon x}] = \exp(\varepsilon\mu \ln P + .5\varepsilon^2\sigma^2(\ln P)^2). \qquad (2)$$

The proof is complete.

APPENDIX 3
PROOF OF VARIANCE'S CONVERGENCE

Given

$$v^2_{\theta(t+1)} = \frac{v^2_{\theta(t)} V^2_{\theta}}{v^2_{\theta(t)} + V^2_{\theta}} < v^2_{\theta(t)},$$

we prove when $t \to \infty$, $v^2_{\theta(t)} \to 0$.

Proof:

At time $t = 0$, we define

$v_0 = v^2_{\theta(0)}$,

$V = V^2_{\theta}$,

$v_0 = kV$.

Then we will obtain the following relationship

$$v_1 = \frac{v_0 V}{v_0 + V} = \frac{kV^2}{(1+k)V} = (\frac{k}{1+k})V,$$

$$v_2 = \frac{v_1 V}{v_1 + V} = \frac{(\frac{k}{1+k})V^2}{(1+\frac{k}{1+k})V} = (\frac{k}{1+2k})V,$$

$$v_3 = \frac{v_2 V}{v_2 + V} = \frac{(\frac{k}{1+2k})V^2}{(1+\frac{k}{1+2k})V} = (\frac{k}{1+3k})V,$$

$$v_t = (\frac{k}{1+tk})V.$$

At time t = t+1,

$$v_{t+1} = \frac{(\frac{k}{1+tk})V^2}{(1+\frac{k}{1+tk})V} = (\frac{k}{1+(t+1)k})V,$$

the proof for $v_t = kV/(1+tk)$ is complete.

When t → ∞,

$$\lim_{t\to\infty} v_t = \lim_{t\to\infty} (\frac{k}{1+tk})V = 0,$$

the proof for $v^2_{\theta(t)} \to 0$ is complete.

APPENDIX 4
PROPERTIES OF THE MANUFACTURER'S OPTIMAL RETAIL PRICE

μ_θ =	c =	ε =
2.0000	1.0000	-1.1000

σ_θ^2 =	P_R =	EP =	Eθ =
0.013	1.84	3.39	2.00
0.016	1.84	3.40	2.00
0.019	1.84	3.41	2.01
0.022	1.84	3.42	2.01
0.025	1.85	3.42	2.01
0.028	1.85	3.43	2.01
0.031	1.85	3.44	2.01
0.034	1.85	3.45	2.01
0.037	1.85	3.46	2.01
0.040	1.85	3.46	2.01
0.043	1.86	3.47	2.01
0.046	1.86	3.48	2.01
0.049	1.86	3.49	2.02
0.052	1.86	3.50	2.02
0.055	1.86	3.51	2.02
0.058	1.86	3.51	2.02
0.061	1.87	3.52	2.02
0.064	1.87	3.53	2.02
0.067	1.87	3.54	2.02
0.070	1.87	3.55	2.02
0.073	1.87	3.56	2.02
0.076	1.88	3.57	2.02
0.079	1.88	3.58	2.02
0.082	1.88	3.59	2.03
0.085	1.88	3.60	2.03
0.088	1.88	3.61	2.03
0.091	1.88	3.61	2.03
0.094	1.89	3.63	2.03
0.097	1.89	3.64	2.03
0.100	1.89	3.64	2.03
0.103	1.89	3.66	2.03
0.106	1.89	3.66	2.03
0.109	1.90	3.68	2.03
0.112	1.90	3.69	2.04
0.115	1.90	3.70	2.04
0.118	1.90	3.71	2.04
0.121	1.90	3.72	2.04

0.124	1.91	3.73	2.04
0.127	1.91	3.74	2.04
0.130	1.91	3.75	2.04

μ_θ = 1.9000

c = 1.0000

ε = -1.1000

σ_θ^2 =	P_R =	EP =	$E\theta$ =
0.013	1.93	3.48	1.90
0.016	1.93	3.49	1.91
0.019	1.93	3.50	1.91
0.022	1.93	3.51	1.91
0.025	1.93	3.52	1.91
0.028	1.94	3.53	1.91
0.031	1.94	3.54	1.91
0.034	1.94	3.55	1.91
0.037	1.94	3.56	1.91
0.040	1.94	3.57	1.91
0.043	1.95	3.58	1.91
0.046	1.95	3.59	1.92
0.049	1.95	3.60	1.92
0.052	1.95	3.61	1.92
0.055	1.96	3.62	1.92
0.058	1.96	3.63	1.92
0.061	1.96	3.64	1.92
0.064	1.96	3.66	1.92
0.067	1.97	3.67	1.92
0.070	1.97	3.68	1.92
0.073	1.97	3.69	1.92
0.076	1.97	3.70	1.93
0.079	1.98	3.71	1.93
0.082	1.98	3.73	1.93
0.085	1.98	3.74	1.93
0.088	1.98	3.75	1.93
0.091	1.99	3.76	1.93
0.094	1.99	3.77	1.93
0.097	1.99	3.78	1.93
0.100	1.99	3.80	1.93
0.103	2.00	3.81	1.94
0.106	2.00	3.82	1.94
0.109	2.00	3.84	1.94
0.112	2.00	3.85	1.94
0.115	2.01	3.86	1.94
0.118	2.01	3.87	1.94
0.121	2.01	3.89	1.94
0.124	2.01	3.90	1.94
0.127	2.02	3.92	1.94
0.130	2.02	3.93	1.95

μ_θ = 1.8000 c = 1.0000 ε = -1.1000

σ_θ^2 =	P_R =	EP =	Eθ =
0.013	2.03	3.59	1.80
0.016	2.03	3.61	1.81
0.019	2.04	3.62	1.81
0.022	2.04	3.63	1.81
0.025	2.04	3.64	1.81
0.028	2.05	3.65	1.81
0.031	2.05	3.66	1.81
0.034	2.05	3.68	1.81
0.037	2.05	3.69	1.81
0.040	2.06	3.70	1.81
0.043	2.06	3.72	1.82
0.046	2.06	3.73	1.82
0.049	2.07	3.74	1.82
0.052	2.07	3.76	1.82
0.055	2.07	3.77	1.82
0.058	2.08	3.78	1.82
0.061	2.08	3.80	1.82
0.064	2.08	3.81	1.82
0.067	2.09	3.83	1.82
0.070	2.09	3.84	1.83
0.073	2.09	3.86	1.83
0.076	2.10	3.87	1.83
0.079	2.10	3.89	1.83
0.082	2.10	3.90	1.83
0.085	2.11	3.91	1.83
0.088	2.11	3.93	1.83
0.091	2.11	3.94	1.83
0.094	2.12	3.96	1.84
0.097	2.12	3.98	1.84
0.100	2.12	3.99	1.84
0.103	2.13	4.01	1.84
0.106	2.13	4.03	1.84
0.109	2.14	4.05	1.84
0.112	2.14	4.06	1.84
0.115	2.14	4.08	1.84
0.118	2.15	4.10	1.85
0.121	2.15	4.12	1.85
0.124	382.61	******	2.17
0.127	382.61	******	2.18
0.130	382.61	******	2.19

μ_θ = 1.7000 c = 1.0000 ε = -1.1000

σ_θ^2 =	P_R =	EP =	Eθ =

0.013	2.17	3.73	1.71
0.016	2.17	3.75	1.71
0.019	2.17	3.76	1.71
0.022	2.18	3.78	1.71
0.025	2.18	3.79	1.71
0.028	2.19	3.81	1.71
0.031	2.19	3.83	1.71
0.034	2.19	3.84	1.71
0.037	2.20	3.86	1.71
0.040	2.20	3.87	1.72
0.043	2.21	3.89	1.72
0.046	2.21	3.91	1.72
0.049	2.22	3.93	1.72
0.052	2.22	3.94	1.72
0.055	2.22	3.96	1.72
0.058	2.23	3.98	1.72
0.061	2.23	4.00	1.72
0.064	2.24	4.02	1.73
0.067	2.24	4.04	1.73
0.070	2.25	4.05	1.73
0.073	2.25	4.07	1.73
0.076	2.26	4.10	1.73
0.079	2.26	4.11	1.73
0.082	2.27	4.13	1.73
0.085	2.27	4.16	1.73
0.088	2.28	4.18	1.74
0.091	2.28	4.20	1.74
0.094	2.29	4.22	1.74
0.097	2.29	4.24	1.74
0.100	2.30	4.27	1.74
0.103	2.31	4.29	1.74
0.106	2.31	4.31	1.74
0.109	382.61	******	2.02
0.112	382.61	******	2.03
0.115	382.61	******	2.04
0.118	999.99	******	2.11
0.121	999.99	******	2.12
0.124	999.99	******	2.13
0.127	999.99	******	2.14
0.130	******	******	2.14

μ_θ = 1.6000

c = 1.0000

ε = -1.1000

σ_θ^2 =	P_R =	EP =	$E\theta$ =
0.013	2.34	3.91	1.61
0.016	2.34	3.93	1.61
0.019	2.35	3.95	1.61
0.022	2.36	3.97	1.61
0.025	2.36	3.99	1.61

0.028	2.37	4.02	1.61
0.031	2.37	4.03	1.61
0.034	2.38	4.06	1.61
0.037	2.39	4.08	1.62
0.040	2.39	4.10	1.62
0.043	2.40	4.12	1.62
0.046	2.41	4.15	1.62
0.049	2.41	4.17	1.62
0.052	2.42	4.20	1.62
0.055	2.43	4.22	1.62
0.058	2.43	4.25	1.63
0.061	2.44	4.27	1.63
0.064	2.45	4.30	1.63
0.067	2.45	4.32	1.63
0.070	2.46	4.35	1.63
0.073	2.47	4.38	1.63
0.076	2.48	4.41	1.63
0.079	2.49	4.44	1.64
0.082	2.49	4.47	1.64
0.085	2.50	4.50	1.64
0.088	2.51	4.53	1.64
0.091	2.52	4.56	1.64
0.094	2.53	4.59	1.64
0.097	382.61	******	1.89
0.100	382.61	******	1.90
0.103	******	******	1.96
0.106	******	******	1.97
0.109	******	******	1.98
0.112	******	******	1.99
0.115	******	******	2.00
0.118	******	******	2.01
0.121	******	******	2.02
0.124	******	******	2.03
0.127	******	******	2.04
0.130	******	******	2.05

μ_θ = 1.5000

c = 1.0000

ε = -1.1000

σ_θ^2 =	P_R =	EP =	$E\theta$ =
0.013	2.57	4.15	1.51
0.016	2.58	4.18	1.51
0.019	2.59	4.21	1.51
0.022	2.60	4.24	1.51
0.025	2.61	4.26	1.51
0.028	2.62	4.30	1.51
0.031	2.63	4.32	1.51
0.034	2.64	4.36	1.52
0.037	2.65	4.39	1.52
0.040	2.66	4.42	1.52
0.043	2.67	4.45	1.52

0.046	2.68	4.49	1.52
0.049	2.69	4.52	1.52
0.052	2.70	4.56	1.53
0.055	2.71	4.59	1.53
0.058	2.72	4.63	1.53
0.061	2.74	4.67	1.53
0.064	2.75	4.71	1.53
0.067	2.76	4.75	1.53
0.070	2.77	4.79	1.54
0.073	2.79	4.84	1.54
0.076	2.80	4.88	1.54
0.079	2.82	4.93	1.54
0.082	382.61	******	1.74
0.085	382.61	******	1.75
0.088	999.99	******	1.80
0.091	******	******	1.81
0.094	******	******	1.82
0.097	999.99	******	1.84
0.100	999.99	******	1.85
0.103	******	******	1.86
0.106	******	******	1.87
0.109	******	******	1.88
0.112	******	******	1.89
0.115	******	******	1.90
0.118	******	******	1.91
0.121	******	******	1.92
0.124	******	******	1.93
0.127	******	******	1.94
0.130	******	******	1.95

σ_θ^2 = 0.0300

c = 1.0000

ε = -1.1000

μ_θ =	P_R =	EP =	$E\theta$ =
1.20	4.80	6.81	1.22
1.25	4.09	5.99	1.27
1.30	3.61	5.44	1.32
1.35	3.26	5.04	1.37
1.40	3.00	4.74	1.42
1.45	2.79	4.50	1.47
1.50	2.63	4.31	1.51
1.55	2.49	4.16	1.56
1.60	2.37	4.03	1.61
1.65	2.27	3.92	1.66
1.70	2.19	3.82	1.71
1.75	2.11	3.74	1.76
1.80	2.05	3.66	1.81
1.85	1.99	3.59	1.86
1.90	1.94	3.54	1.91

σ_θ^2 =	c =	ε =
0.0500	1.0000	-1.1000

μ_θ =	P_R =	EP =	Eθ =
1.20	999.99	******	1.37
1.25	******	******	1.42
1.30	3.87	6.08	1.33
1.35	3.44	5.51	1.38
1.40	3.12	5.08	1.43
1.45	2.88	4.78	1.48
1.50	2.69	4.53	1.52
1.55	2.54	4.34	1.57
1.60	2.42	4.18	1.62
1.65	2.31	4.05	1.67
1.70	2.22	3.93	1.72
1.75	2.14	3.83	1.77
1.80	2.07	3.75	1.82
1.85	2.01	3.67	1.87
1.90	1.95	3.61	1.92

μ_θ =	σ_θ^2 =	c =
1.8000	0.0500	1.0000

ε =	P_R =	EP =	Eθ =
-1.02	2.26	4.41	1.82
-1.04	2.21	4.22	1.82
-1.06	2.16	4.05	1.82
-1.08	2.11	3.89	1.82
-1.10	2.07	3.75	1.82
-1.12	2.03	3.62	1.82
-1.14	1.99	3.50	1.82
-1.16	1.96	3.39	1.82
-1.18	1.93	3.29	1.82
-1.20	1.90	3.20	1.82
-1.22	1.87	3.12	1.82
-1.24	1.84	3.04	1.82
-1.26	1.82	2.96	1.81
-1.28	1.80	2.89	1.81
-1.30	1.77	2.83	1.81

μ_θ =	σ_θ^2 =	c =
1.6000	0.0500	1.0000

ε =	P_R =	EP =	Eθ =
-1.02	2.72	5.10	1.63
-1.04	2.63	4.82	1.62

-1.06	2.56	4.59	1.62
-1.08	2.48	4.37	1.62
-1.10	2.42	4.18	1.62
-1.12	2.35	4.01	1.62
-1.14	2.30	3.85	1.62
-1.16	2.25	3.71	1.62
-1.18	2.20	3.59	1.62
-1.20	2.16	3.47	1.62
-1.22	2.12	3.36	1.62
-1.24	2.08	3.27	1.62
-1.26	2.04	3.18	1.62
-1.28	2.01	3.09	1.62
-1.30	1.98	3.01	1.62

$\mu_\theta =$	$\sigma_\theta^2 =$	$\varepsilon =$
1.8000	0.0500	-1.1000

c =	P_R =	EP =	$E\theta =$
1.00	2.07	3.75	1.82
1.10	2.28	4.49	1.82
1.20	2.50	5.30	1.82
1.30	2.71	6.18	1.82
1.40	2.93	7.13	1.83
1.50	3.15	8.14	1.83
1.60	3.37	9.22	1.83
1.70	3.58	10.35	1.83
1.80	3.80	11.57	1.83
1.90	4.02	12.84	1.83
2.00	4.24	14.19	1.84
2.10	4.46	15.60	1.84
2.20	4.68	17.08	1.84
2.30	4.90	18.62	1.84
2.40	5.12	20.24	1.84

$\mu_\theta =$	$\sigma_\theta^2 =$	$\varepsilon =$
1.6000	0.0500	-1.1000

c =	P_R =	EP =	$E\theta =$
1.00	2.42	4.18	1.62
1.10	2.67	4.93	1.62
1.20	2.93	5.74	1.63
1.30	3.18	6.59	1.63
1.40	3.44	7.51	1.63
1.50	3.70	8.47	1.63
1.60	3.96	9.49	1.63
1.70	4.23	10.57	1.64
1.80	4.49	11.69	1.64
1.90	4.75	12.87	1.64
2.00	5.02	14.09	1.64
2.10	5.29	15.38	1.64

2.20	5.55	16.71	1.64
2.30	5.82	18.10	1.64
2.40	6.09	19.53	1.65

APPENDIX 5
PROPERTIES OF THE RETAILER'S OPTIMAL MARKET PRICE

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.9000	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	2.38	2.27	0.95
0.120	2.22	2.13	0.95
0.130	2.09	2.01	0.95
0.140	1.99	1.92	0.95
0.150	1.90	1.84	0.95
0.160	1.83	1.77	0.95
0.170	1.76	1.71	0.95
0.180	1.71	1.66	0.95
0.190	1.67	1.62	0.95
0.200	1.63	1.58	0.95
0.210	1.58	1.55	0.95
0.220	1.55	1.52	0.95
0.230	1.52	1.49	0.95
0.240	1.50	1.47	0.95
0.250	1.48	1.45	0.95

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.8500	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	3.54	3.20	20.92
0.120	3.21	2.93	20.92
0.130	2.95	2.71	30.92
0.140	2.74	2.53	30.92
0.150	2.58	2.39	30.92
0.160	2.43	2.27	30.92
0.170	2.32	2.17	30.92
0.180	2.22	2.08	40.92
0.190	2.13	2.01	40.92
0.200	2.05	1.94	40.92
0.210	1.99	1.88	40.92
0.220	1.92	1.83	40.92
0.230	1.87	1.78	50.92
0.240	1.83	1.75	50.92
0.250	1.78	1.70	50.92

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.8000	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	5.15	4.30	0.89
0.120	4.56	3.87	0.89
0.130	4.10	3.52	0.89
0.140	3.74	3.25	0.89
0.150	3.45	3.02	0.89
0.160	3.21	2.83	0.89
0.170	3.01	2.68	0.89
0.180	2.84	2.54	0.89
0.190	2.70	2.43	0.89
0.200	2.58	2.33	0.89
0.210	2.47	2.25	0.89
0.220	2.37	2.17	0.90
0.230	2.29	2.10	0.90
0.240	2.22	2.04	0.90
0.250	2.15	1.98	0.90

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.7500	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	7.24	5.47	0.86
0.120	6.29	4.86	0.86
0.130	5.56	4.38	0.86
0.140	5.00	4.01	0.86
0.150	4.54	3.69	0.86
0.160	4.17	3.44	0.86
0.170	3.86	3.22	0.86
0.180	3.61	3.04	0.87
0.190	3.39	2.88	0.87
0.200	3.21	2.74	0.87
0.210	3.04	2.62	0.87
0.220	2.90	2.52	0.87
0.230	2.78	2.43	0.87
0.240	2.67	2.34	0.87
0.250	2.57	2.27	0.87

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.7000	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	9.80	6.58	0.83
0.120	8.42	5.84	0.83
0.130	7.36	5.24	0.83
0.140	6.53	4.76	0.83

0.150	5.87	4.36	0.83
0.160	5.34	4.04	0.83
0.170	4.89	3.76	0.83
0.180	4.53	3.54	0.84
0.190	4.21	3.33	0.84
0.200	3.95	3.16	0.84
0.210	3.72	3.01	0.84
0.220	3.53	2.88	0.84
0.230	3.35	2.76	0.84
0.240	3.20	2.65	0.84
0.250	3.06	2.56	0.84

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.6500	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	12.68	7.43	0.79
0.120	10.86	6.63	0.79
0.130	9.44	5.97	0.80
0.140	8.33	5.43	0.80
0.150	7.42	4.98	0.80
0.160	6.70	4.60	0.80
0.170	6.09	4.27	0.80
0.180	5.60	4.00	0.81
0.190	5.17	3.76	0.81
0.200	4.81	3.55	0.81
0.210	4.51	3.38	0.81
0.220	4.24	3.22	0.81
0.230	4.01	3.08	0.81
0.240	3.81	2.95	0.81
0.250	3.63	2.84	0.81

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.6000	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	15.58	7.87	0.75
0.120	13.43	7.12	0.76
0.130	11.67	6.47	0.76
0.140	10.29	5.92	0.76
0.150	9.14	5.45	0.77
0.160	8.22	5.05	0.77
0.170	7.45	4.70	0.77
0.180	6.81	4.40	0.77
0.190	6.26	4.14	0.77
0.200	5.80	3.91	0.78
0.210	5.40	3.71	0.78
0.220	5.06	3.53	0.78

0.230	4.76	3.37	0.78
0.240	4.49	3.23	0.78
0.250	4.27	3.11	0.78

$\mu_\rho =$	$\varepsilon =$	$k_2 =$
0.5500	-1.1000	1.0000

$\sigma_\rho^2 =$	$P =$	$EP_R =$	$E\rho =$
0.110	18.02	7.77	0.71
0.120	15.75	7.19	0.72
0.130	13.86	6.65	0.72
0.140	12.27	6.17	0.73
0.150	10.93	5.72	0.73
0.160	9.84	5.34	0.73
0.170	8.91	5.00	0.74
0.180	8.12	4.70	0.74
0.190	7.46	4.43	0.74
0.200	6.88	4.19	0.74
0.210	6.39	3.98	0.74
0.220	5.95	3.79	0.75
0.230	5.58	3.62	0.75
0.240	5.26	3.47	0.75
0.250	4.97	3.33	0.75

$\sigma_\rho^2 =$	$\varepsilon =$	$k_2 =$
0.1200	-1.1000	1.0000

$\mu_\rho =$	$P =$	$EP_R =$	$E\rho =$
0.60	13.43	7.12	0.76
0.62	12.40	6.97	0.77
0.64	11.36	6.75	0.79
0.66	10.35	6.49	0.80
0.68	9.37	6.18	0.81
0.70	8.42	5.84	0.83
0.72	7.52	5.46	0.84
0.74	6.68	5.06	0.85
0.76	5.91	4.66	0.87
0.78	5.20	4.26	0.88
0.80	4.56	3.87	0.89
0.82	3.97	3.47	0.90
0.84	3.45	3.11	0.91
0.86	2.99	2.76	0.93

$\sigma_\rho^2 =$	$\varepsilon =$	$k_2 =$
0.1500	-1.1000	1.0000

$\mu_\rho =$	$P =$	$EP_R =$	$E\rho =$
0.60	9.14	5.45	0.77

0.62	8.45	5.28	0.78
0.64	7.75	5.08	0.79
0.66	7.09	4.86	0.81
0.68	6.47	4.62	0.82
0.70	5.87	4.37	0.83
0.72	5.31	4.10	0.85
0.74	4.79	3.83	0.86
0.76	4.30	3.55	0.87
0.78	3.85	3.28	0.88
0.80	3.45	3.02	0.89
0.82	3.07	2.76	0.90
0.84	2.73	2.51	0.92
0.86	2.43	2.28	0.93
0.88	2.15	2.05	0.94

$\mu_\rho =$	$\sigma_\rho^2 =$	$k_2 =$
0.8500	0.1200	1.0000

$\varepsilon =$	$P =$	$EP_R =$	$E\rho =$
-1.02	3.43	3.12	0.92
-1.04	3.38	3.07	0.92
-1.06	3.32	3.03	0.92
-1.08	3.27	2.98	0.92
-1.10	3.21	2.93	0.92
-1.12	3.16	2.88	0.92
-1.14	3.11	2.83	0.92
-1.16	3.06	2.79	0.92
-1.18	3.01	2.75	0.92
-1.20	2.97	2.71	0.92
-1.22	2.92	2.67	0.91
-1.24	2.88	2.62	0.91
-1.26	2.84	2.59	0.91
-1.28	2.79	2.55	0.91
-1.30	2.75	2.51	0.91

$\mu_\rho =$	$\sigma_\rho^2 =$	$k_2 =$
0.8000	0.1500	1.0000

$\varepsilon =$	$P =$	$EP_R =$	$E\rho =$
-1.02	3.72	3.26	0.90
-1.04	3.65	3.19	0.90
-1.06	3.58	3.13	0.90
-1.08	3.52	3.08	0.89
-1.10	3.45	3.02	0.89
-1.12	3.39	2.97	0.89
-1.14	3.32	2.91	0.89
-1.16	3.26	2.86	0.89
-1.18	3.20	2.80	0.89
-1.20	3.15	2.76	0.89
-1.22	3.09	2.71	0.88

-1.24	3.04	2.67	0.88
-1.26	2.98	2.62	0.88
-1.28	2.93	2.58	0.88
-1.30	2.88	2.54	0.88

$\mu_\rho =$ 0.8500

$\sigma_\rho^2 =$ 0.1200

$\varepsilon =$ -1.1000

$k_2 =$	$P =$	$EP_R =$	$E\rho =$
0.85	2.74	2.13	0.91
0.87	2.81	2.23	0.91
0.89	2.87	2.33	0.91
0.91	2.94	2.44	0.91
0.93	3.00	2.55	0.92
0.95	3.06	2.65	0.92
0.97	3.12	2.76	0.92
0.99	3.18	2.87	0.92
1.01	3.24	2.98	0.92
1.03	3.30	3.10	0.92
1.05	3.35	3.21	0.92
1.07	3.41	3.32	0.92
1.09	3.46	3.43	0.92
1.11	3.52	3.55	0.93
1.13	3.57	3.67	0.93

$\mu_\rho =$ 0.8000

$\sigma_\rho^2 =$ 0.1200

$\varepsilon =$ -1.1000

$k_2 =$	$P =$	$EP_R =$	$E\rho =$
0.85	3.02	2.25	0.88
0.87	3.08	2.35	0.88
0.89	3.15	2.46	0.89
0.91	3.20	2.55	0.89
0.93	3.26	2.66	0.89
0.95	3.31	2.76	0.89
0.97	3.36	2.86	0.89
0.99	3.42	2.97	0.89
1.01	3.48	3.07	0.89
1.03	3.53	3.18	0.89
1.05	3.57	3.28	0.90
1.07	3.62	3.39	0.90
1.09	3.67	3.51	0.90
1.11	3.72	3.61	0.90
1.13	3.76	3.72	0.90

APPENDIX 6

BARGAINING RESULT WHEN THE MARKET DEMAND ELASTICITY $\varepsilon = -1.12$

$a =$	$c =$	$\varepsilon =$	$k_1 =$	$k_2 =$
10000.0	1.00	-1.12	1.00	1.00

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

OBS =	$P_R =$	$EP =$	$P =$	$EP_R =$
1	2.24	3.83	3.08	2.70
2	2.36	3.95	3.34	2.88
3	2.43	4.03	3.56	3.02
4	2.48	4.08	3.75	3.14
5	2.51	4.11	3.94	3.26
6	2.52	4.11	4.14	3.37
7	2.51	4.10	4.34	3.49
8	2.49	4.08	4.56	3.60
9	2.46	4.04	4.79	3.72
10	2.42	3.99	5.05	3.85
11	2.37	3.94	5.32	3.98
12	2.31	3.88	5.61	4.10
13	2.25	3.81	5.92	4.23
14	2.19	3.74	6.25	4.35
15	2.13	3.68	6.59	4.47
16	2.07	3.61	6.94	4.58
17	2.01	3.55	7.29	4.68
18	1.96	3.48	7.62	4.76
19	1.90	3.42	7.99	4.85
20	1.85	3.37	8.35	4.92
21	1.80	3.31	8.68	4.96
22	1.75	3.26	9.02	5.01
23	1.71	3.21	9.33	5.03
24	1.67	3.17	9.65	5.06
25	1.63	3.12	9.93	5.06
26	1.59	3.09	10.21	5.06
27	1.56	3.05	10.48	5.05
28	1.53	3.02	10.71	5.02
29	1.50	2.98	10.92	4.99
30	1.47	2.95	11.14	4.96

OBS =	q =	π_M =	π_R =	π_T =
1	2833.4	3508.2	2394.6	5902.7
2	2587.4	3510.0	2554.2	6064.1
3	2411.1	3458.9	2716.4	6175.2
4	2272.4	3368.8	2890.9	6259.6
5	2150.5	3245.5	3085.8	6331.2
6	2037.2	3090.5	3305.0	6395.5
7	1930.8	2912.1	3541.5	6453.6
8	1829.0	2720.8	3786.1	6506.9
9	1731.2	2521.3	4034.5	6555.8
10	1630.1	2307.0	4296.5	6603.6
11	1537.7	2100.1	4544.6	6644.7
12	1449.2	1901.5	4779.9	6681.4
13	1364.2	1711.6	5002.3	6713.9
14	1284.5	1532.9	5208.8	6741.6
15	1210.8	1371.5	5393.1	6764.7
16	1142.8	1225.8	5557.6	6783.5
17	1080.8	1095.3	5703.0	6798.2
18	1027.8	982.2	5826.7	6808.9
19	975.3	878.1	5939.4	6817.5
20	928.5	787.0	6036.4	6823.4
21	889.2	709.6	6117.3	6826.8
22	851.4	639.1	6189.6	6828.8
23	820.0	579.5	6249.8	6829.3
24	789.5	525.4	6303.5	6828.8
25	764.7	479.8	6347.8	6827.6
26	741.2	438.6	6387.2	6825.8
27	719.9	401.9	6421.6	6823.5
28	702.4	370.4	6450.7	6821.1
29	687.0	342.5	6476.2	6818.7
30	672.3	317.2	6498.8	6816.0

OBS	μ_θ =	v_θ^2 =	σ_θ^2 =	$\theta_{(t)}$ =
1	1.6500	0.0100	0.0400	1.3976
2	1.5869	0.0075	0.0375	1.4082
3	1.5512	0.0060	0.0360	1.4274
4	1.5305	0.0050	0.0350	1.4550
5	1.5197	0.0043	0.0343	1.4916
6	1.5162	0.0038	0.0338	1.5389
7	1.5188	0.0033	0.0333	1.5969
8	1.5266	0.0030	0.0330	1.6644
9	1.5391	0.0027	0.0327	1.7424
10	1.5560	0.0025	0.0325	1.8367
11	1.5776	0.0023	0.0323	1.9414
12	1.6036	0.0021	0.0321	2.0577
13	1.6339	0.0020	0.0320	2.1877
14	1.6685	0.0019	0.0319	2.3329
15	1.7076	0.0018	0.0318	2.4889
16	1.7510	0.0017	0.0317	2.6573
17	1.7987	0.0016	0.0316	2.8385
18	1.8507	0.0015	0.0315	3.0287
19	1.9068	0.0014	0.0314	3.2370

20	1.9672	0.0014	0.0314	3.4569
21	2.0320	0.0013	0.0313	3.6830
22	2.1008	0.0012	0.0313	3.9276
23	2.1739	0.0012	0.0312	4.1772
24	2.2509	0.0012	0.0312	4.4442
25	2.3321	0.0011	0.0311	4.7136
26	2.4172	0.0011	0.0311	4.9984
27	2.5062	0.0010	0.0310	5.2967
28	2.5992	0.0010	0.0310	5.5986
29	2.6960	0.0010	0.0310	5.9118
30	2.7965	0.0009	0.0309	6.2363

OBS =	$\mu_\rho =$	$v_\rho^2 =$	$\sigma_\rho^2 =$	$\rho_{(t)} =$
1	0.7800	0.0300	0.1800	0.7155
2	0.7693	0.0250	0.1750	0.7101
3	0.7608	0.0214	0.1714	0.7006
4	0.7533	0.0187	0.1688	0.6873
5	0.7459	0.0167	0.1667	0.6704
6	0.7384	0.0150	0.1650	0.6498
7	0.7303	0.0136	0.1636	0.6262
8	0.7217	0.0125	0.1625	0.6008
9	0.7124	0.0115	0.1615	0.5739
10	0.7025	0.0107	0.1607	0.5445
11	0.6919	0.0100	0.1600	0.5151
12	0.6809	0.0094	0.1594	0.4860
13	0.6694	0.0088	0.1588	0.4571
14	0.6576	0.0083	0.1583	0.4287
15	0.6456	0.0079	0.1579	0.4018
16	0.6334	0.0075	0.1575	0.3763
17	0.6211	0.0071	0.1571	0.3523
18	0.6089	0.0068	0.1568	0.3302
19	0.5968	0.0065	0.1565	0.3089
20	0.5848	0.0062	0.1563	0.2893
21	0.5730	0.0060	0.1560	0.2715
22	0.5614	0.0058	0.1558	0.2546
23	0.5500	0.0056	0.1556	0.2394
24	0.5389	0.0054	0.1554	0.2250
25	0.5281	0.0052	0.1552	0.2122
26	0.5176	0.0050	0.1550	0.2001
27	0.5073	0.0048	0.1548	0.1888
28	0.4974	0.0047	0.1547	0.1786
29	0.4877	0.0045	0.1545	0.1692
30	0.4784	0.0044	0.1544	0.1604

APPENDIX 7

BARGAINING RESULT WHEN THE MARKET DEMAND ELASTICITY $\varepsilon = -1.15$

$a =$	$c =$	$\varepsilon =$	$k_1 =$	$k_2 =$
10000.0	1.00	-1.15	1.00	1.00

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

OBS	$P_R =$	$EP =$	$P =$	$EP_R =$
1	2.17	3.62	3.02	2.64
2	2.26	3.72	3.29	2.82
3	2.31	3.76	3.54	2.98
4	2.33	3.78	3.79	3.13
5	2.32	3.77	4.05	3.27
6	2.30	3.74	4.32	3.42
7	2.25	3.69	4.63	3.57
8	2.20	3.64	4.95	3.71
9	2.14	3.58	5.29	3.85
10	2.08	3.51	5.64	3.98
11	2.01	3.44	6.00	4.10
12	1.95	3.37	6.36	4.20
13	1.88	3.30	6.71	4.28
14	1.82	3.24	7.06	4.35
15	1.77	3.18	7.39	4.39
16	1.71	3.13	7.70	4.41
17	1.67	3.08	8.00	4.43
18	1.62	3.03	8.27	4.43
19	1.58	2.99	8.52	4.41
20	1.54	2.94	8.75	4.38
21	1.51	2.91	8.95	4.34
22	1.47	2.88	9.11	4.29
23	1.44	2.84	9.27	4.24
24	1.42	2.82	9.40	4.19
25	1.39	2.79	9.52	4.13
26	1.37	2.77	9.62	4.06
27	1.35	2.75	9.71	4.00
28	1.33	2.73	9.78	3.94
29	1.31	2.71	9.84	3.87
30	1.30	2.69	9.89	3.81

OBS =	q =	π_M =	π_R =	π_T =
1	2808.7	3276.9	2388.0	5664.9
2	2541.4	3205.0	2617.4	5822.4
3	2336.8	3061.6	2874.2	5935.9
4	2162.7	2874.0	3152.9	6026.9
5	2002.6	2648.7	3456.5	6105.2
6	1856.3	2406.8	3764.9	6171.7
7	1715.8	2152.2	4078.0	6230.2
8	1590.1	1912.0	4365.4	6277.4
9	1473.6	1684.2	4632.0	6316.2
10	1368.8	1476.4	4870.1	6346.4
11	1274.6	1290.6	5078.6	6369.2
12	1192.0	1129.3	5256.0	6385.3
13	1119.4	989.8	5406.3	6396.0
14	1056.0	870.2	5532.3	6402.5
15	1002.8	769.3	5636.2	6405.6
16	956.1	682.8	5723.6	6406.3
17	915.6	609.2	5796.2	6405.4
18	880.5	546.0	5857.3	6403.3
19	850.6	492.3	5908.2	6400.4
20	825.5	445.9	5951.4	6397.2
21	804.5	406.9	5987.1	6394.0
22	787.6	373.1	6017.9	6391.0
23	772.2	343.1	6044.8	6387.9
24	759.8	317.8	6067.4	6385.2
25	748.9	294.7	6087.9	6382.7
26	739.9	274.5	6106.0	6380.4
27	732.5	257.0	6121.5	6378.5
28	726.7	240.9	6136.0	6376.9
29	721.4	226.9	6148.5	6375.5
30	717.2	214.2	6160.1	6374.3

OBS =	μ_θ =	v_θ^2 =	σ_θ^2 =	$\theta_{(t)}$ =
1	1.6500	0.0100	0.0400	1.4281
2	1.5945	0.0075	0.0375	1.4601
3	1.5676	0.0060	0.0360	1.5098
4	1.5580	0.0050	0.0350	1.5750
5	1.5604	0.0043	0.0343	1.6594
6	1.5728	0.0038	0.0338	1.7613
7	1.5937	0.0033	0.0333	1.8857
8	1.6229	0.0030	0.0330	2.0251
9	1.6595	0.0027	0.0327	2.1847
10	1.7033	0.0025	0.0325	2.3634
11	1.7540	0.0023	0.0323	2.5611
12	1.8117	0.0021	0.0321	2.7750
13	1.8759	0.0020	0.0320	3.0058
14	1.9465	0.0019	0.0319	3.2523
15	2.0233	0.0018	0.0318	3.5122
16	2.1061	0.0017	0.0317	3.7879
17	2.1946	0.0016	0.0316	4.0758
18	2.2886	0.0015	0.0315	4.3794
19	2.3882	0.0014	0.0314	4.6932

20	2.4930	0.0014	0.0314	5.0222
21	2.6029	0.0013	0.0313	5.3534
22	2.7175	0.0012	0.0313	5.6980
23	2.8368	0.0012	0.0312	6.0572
24	2.9606	0.0012	0.0312	6.4137
25	3.0885	0.0011	0.0311	6.7909
26	3.2207	0.0011	0.0311	7.1764
27	3.3571	0.0010	0.0310	7.5584
28	3.4972	0.0010	0.0310	7.9618
29	3.6412	0.0010	0.0310	8.3590
30	3.7886	0.0009	0.0309	8.7683

OBS =	$\mu_\rho =$	$v_\rho^2 =$	$\sigma_\rho^2 =$	$\rho_{(t)} =$
1	0.7800	0.0300	0.1800	0.7002
2	0.7667	0.0250	0.1750	0.6849
3	0.7550	0.0214	0.1714	0.6623
4	0.7434	0.0187	0.1688	0.6349
5	0.7314	0.0167	0.1667	0.6026
6	0.7185	0.0150	0.1650	0.5678
7	0.7048	0.0136	0.1636	0.5303
8	0.6903	0.0125	0.1625	0.4938
9	0.6751	0.0115	0.1615	0.4577
10	0.6596	0.0107	0.1607	0.4231
11	0.6438	0.0100	0.1600	0.3905
12	0.6280	0.0094	0.1594	0.3604
13	0.6123	0.0088	0.1588	0.3327
14	0.5967	0.0083	0.1583	0.3075
15	0.5815	0.0079	0.1579	0.2847
16	0.5667	0.0075	0.1575	0.2640
17	0.5523	0.0071	0.1571	0.2454
18	0.5383	0.0068	0.1568	0.2283
19	0.5248	0.0065	0.1565	0.2131
20	0.5118	0.0062	0.1563	0.1991
21	0.4993	0.0060	0.1560	0.1868
22	0.4873	0.0058	0.1558	0.1755
23	0.4758	0.0056	0.1556	0.1651
24	0.4647	0.0054	0.1554	0.1559
25	0.4540	0.0052	0.1552	0.1473
26	0.4438	0.0050	0.1550	0.1393
27	0.4340	0.0048	0.1548	0.1323
28	0.4245	0.0047	0.1547	0.1256
29	0.4155	0.0045	0.1545	0.1196
30	0.4068	0.0044	0.1544	0.1140

APPENDIX 8
BARGAINING PROCESS THAT THE MANUFACTURER QUOTES THE REATIL PRICE FIRST

a =	c =	ε =	k_1 =	k_2 =
10000.0	1.00	-1.10	1.00	1.00

μ_θ =	v_θ^2 =	V_θ^2 =	μ_ρ =	v_ρ^2 =	V_ρ^2 =
1.650	0.010	0.030	0.780	0.030	0.150

OBS =	P_R =	EP =	P =	EP_R =
1	2.29	3.98	3.38	2.91
2	2.38	4.07	3.60	3.06
3	2.43	4.13	3.83	3.20
4	2.46	4.15	4.07	3.35
5	2.46	4.15	4.34	3.51
6	2.44	4.12	4.64	3.67
7	2.40	4.07	4.98	3.86
8	2.34	4.01	5.37	4.04
9	2.28	3.94	5.79	4.24
10	2.21	3.86	6.24	4.43
11	2.14	3.77	6.74	4.63
12	2.06	3.68	7.25	4.80
13	1.98	3.59	7.79	4.97
14	1.91	3.51	8.34	5.12
15	1.85	3.44	8.88	5.24
16	1.78	3.36	9.43	5.35
17	1.72	3.30	9.95	5.43
18	1.67	3.23	10.46	5.49
19	1.62	3.18	10.98	5.55
20	1.57	3.13	11.44	5.56

OBS =	q =	π_M =	π_R =	π_T =
1	2622.5	3383.0	2848.8	6231.9
2	2445.1	3369.7	2983.3	6353.0
3	2283.6	3268.2	3191.9	6460.1
4	2135.4	3108.1	3447.0	6555.1
5	1988.8	2897.5	3748.2	6645.7
6	1848.9	2653.9	4074.6	6728.5
7	1708.3	2386.3	4421.3	6807.6
8	1575.5	2117.7	4760.3	6878.0
9	1449.9	1855.7	5084.3	6940.0
10	1334.0	1613.9	5378.8	6992.6
11	1225.0	1390.3	5647.0	7037.4
12	1131.8	1198.3	5873.0	7071.3

13	1044.9	1028.7	6070.2	7098.9
14	970.1	885.3	6233.6	7118.8
15	905.5	765.7	6367.2	7132.9
16	847.4	661.8	6480.9	7142.7
17	798.9	577.1	6571.5	7148.5
18	755.7	503.9	6647.7	7151.7
19	716.6	441.7	6711.0	7152.7
20	685.1	391.2	6760.9	7152.1

OBS =	$\mu_\theta =$	$v_\theta^2 =$	$\sigma_\theta^2 =$	$\theta_{(t)} =$
1	1.6500	0.0100	0.0400	1.4686
2	1.6046	0.0075	0.0375	1.4780
3	1.5793	0.0060	0.0360	1.5113
4	1.5680	0.0050	0.0350	1.5624
5	1.5672	0.0043	0.0343	1.6333
6	1.5755	0.0038	0.0338	1.7240
7	1.5920	0.0033	0.0333	1.8377
8	1.6165	0.0030	0.0330	1.9720
9	1.6488	0.0027	0.0327	2.1302
10	1.6890	0.0025	0.0325	2.3096
11	1.7367	0.0023	0.0323	2.5166
12	1.7924	0.0021	0.0321	2.7429
13	1.8558	0.0020	0.0320	2.9959
14	1.9270	0.0019	0.0319	3.2707
15	2.0061	0.0018	0.0318	3.5630
16	2.0926	0.0017	0.0317	3.8876
17	2.1870	0.0016	0.0316	4.2256
18	2.2890	0.0015	0.0315	4.5954
19	2.3988	0.0014	0.0314	4.9903
20	2.5166	0.0014	0.0314	5.3953

OBS =	$\mu_\rho =$	$v_\rho^2 =$	$\sigma_\rho^2 =$	$\rho_{(t)} =$
1	0.7710	0.0250	0.1750	0.7257
2	0.7625	0.0214	0.1714	0.7120
3	0.7539	0.0187	0.1688	0.6938
4	0.7445	0.0167	0.1667	0.6691
5	0.7341	0.0150	0.1650	0.6404
6	0.7225	0.0136	0.1636	0.6062
7	0.7097	0.0125	0.1625	0.5697
8	0.6959	0.0115	0.1615	0.5303
9	0.6813	0.0107	0.1607	0.4905
10	0.6660	0.0100	0.1600	0.4517
11	0.6502	0.0094	0.1594	0.4142
12	0.6342	0.0088	0.1588	0.3783
13	0.6182	0.0083	0.1583	0.3460
14	0.6023	0.0079	0.1579	0.3158
15	0.5866	0.0075	0.1575	0.2890
16	0.5713	0.0071	0.1571	0.2643
17	0.5563	0.0068	0.1568	0.2423
18	0.5418	0.0065	0.1565	0.2224
19	0.5278	0.0062	0.1563	0.2045
20	0.5142	0.0060	0.1560	0.1885

APPENDIX 9
BARGAINING PROCESS THAT THE RETAILER QUOTES THE MARKET PRICE FIRST

$a =$	$c =$	$\varepsilon =$	$k_1 =$	$k_2 =$
10000.0	1.00	-1.10	1.00	1.00

$\mu_\theta =$	$v_\theta^2 =$	$V_\theta^2 =$	$\mu_\rho =$	$v_\rho^2 =$	$V_\rho^2 =$
1.650	0.010	0.030	0.780	0.030	0.150

OBS =	$P_R =$	EP=	P =	$EP_R =$
1	2.43	4.13	3.13	2.74
2	2.56	4.27	3.24	2.82
3	2.69	4.42	3.26	2.84
4	2.84	4.60	3.21	2.82
5	3.04	4.83	3.06	2.72
6	3.33	5.16	2.79	2.53
7	3.82	5.73	2.36	2.21
8	4.90	7.02	1.70	1.66
9	*****	*****	1.10	1.11
10	*****	*****	1.10	1.80

OBS =	q =	$\pi_M =$	$\pi_R =$	$\pi_T =$
1	2848.4	4070.1	2002.7	6072.7
2	2746.2	4275.7	1869.6	6145.3
3	2723.6	4601.2	1560.0	6161.2
4	2770.2	5106.9	1021.4	6128.3
5	2920.3	5971.4	49.6	6021.0
6	3232.6	7544.2	-1752.5	5791.7
7	3884.0	10959.1	-5666.9	5292.2
8	5580.5	21782.9	-17879.9	3903.0
9	9004.7	********	********	900.0
10	9004.7	********	********	900.0

OBS =	$\mu_\theta =$	$v_\theta^2 =$	$\sigma_\theta^2 =$	$\theta_{(t)} =$
1	1.5820	0.0075	0.0375	1.3779
2	1.5304	0.0060	0.0360	1.3239
3	1.4852	0.0050	0.0350	1.2594
4	1.4415	0.0043	0.0343	1.1796
5	1.3952	0.0038	0.0338	1.0708
6	1.3426	0.0033	0.0333	0.9221
7	1.2798	0.0030	0.0330	0.7140

8	1.1994	0.0027	0.0327	0.3955
9	1.1044	0.0025	0.0325	0.0599
10	1.0205	0.0023	0.0323	0.0138

OBS =	$\mu_\rho =$	$v_\rho^2 =$	$\sigma_\rho^2 =$	$\rho_{(t)} =$
1	0.7800	0.0300	0.1800	0.7773
2	0.7796	0.0250	0.1750	0.7991
3	0.7823	0.0214	0.1714	0.8367
4	0.7891	0.0187	0.1688	0.8955
5	0.8010	0.0167	0.1667	0.9950
6	0.8204	0.0150	0.1650	1.1729
7	0.8524	0.0136	0.1636	1.5594
8	0.9113	0.0125	0.1625	2.9983
9	1.0719	0.0115	0.1615	72.4765
10	6.1722	0.0107	0.1607	72.4765

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